Robust engineering of universal Gaussian cluster states for continuous variable measurement-based quantum computation

Stefano Zippilli, David Vitali

School of Science and Technology, Physics Division,
University of Camerino, Italy

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Outline

• We mathematically specify the recipe for generating **any given CV (continuous variable) cluster state** with a multimode squeezing interaction

• We show that a large class of pure entangled CV Gaussian states, including cluster states, can be **robustly generated** as **unique steady state of a dissipative dynamics,** based on minimal resources: **i) single-site squeezed** driving: **ii) passive bilinear** interactions.

CV quantum computation

- Information encoded in **bosonic modes**
- natural environment for **measurement-based quantum computation**: cluster states as main resource
- “Clifford” gates are obtained with Gaussian operations and homodyne measurements:
- A nonlinear element (gate or nonlinear encoding, e.g., GKP states) for non-Clifford operations
- Recent promising progress: Xanadu results, first demonstrations by Furusawa and U. Andersen groups
- Not only CV **optical modes**, but also **microwave cavity arrays** as platforms (see S. Gasparinetti talk)

CV cluster states are the main resource for measurement based quantum computation (MBQC). Potentially useful for sensing and state discrimination

Ideal CV cluster states **$|\Psi\rangle$** are the zero eigenstates of the nullifiers, $x_j | \Psi\rangle = 0$.

$$x_j = -\mathrm{i}\left(b_j\,\mathrm{e}^{\mathrm{i}\,\theta_j} - b_j^\dagger\,\mathrm{e}^{-\mathrm{i}\,\theta_j}\right) - \sum_{k=1}^{N} \mathcal{A}_{j,k}\left(b_k\,\mathrm{e}^{\mathrm{i}\,\theta_k} + b_k^\dagger\,\mathrm{e}^{-\mathrm{i}\,\theta_k}\right)$$

$[b_j, b_k^+] = \delta_{jk}$

A_{jk} = adjacency matrix (real symmetric), defining the cluster state.

They have infinite energy and one has to consider only physically realizable **finitely squeezed Gaussian approximations**

It is known that any multi-mode squeezing transformation generates a cluster state. It is useful to specify also the reverse path: given a cluster state with generic adjacency matrix *A*, what is the class of multi-mode squeezing transformations generating it ?

The recipe: given A_{jk}, the corresponding cluster state $|\Psi\rangle$ is obtained as

$$|\Psi\rangle = U\,|0\rangle \qquad U = e^{-i\frac{z}{2}\sum_{j,k=1}^{N}\left(\mathcal{Z}_{j,k}b_j^\dagger b_k^\dagger + \mathcal{Z}_{j,k}^\dagger b_j b_k\right)}$$

with the symmetric matrix ***Z*** written as a polar decomposition

$$\mathcal{Z} = \mathcal{P}\,\mathcal{U} \qquad \mathcal{U} = -i\,e^{-i\Theta}\,\frac{\mathcal{A} - i\,\mathbb{1}}{\mathcal{A} + i\,\mathbb{1}}\,e^{-i\Theta} \qquad \Theta = \mathrm{diag}(\theta_1, \theta_2, \ldots)$$

and **with arbitrary Hermitian positive *P*** such that ***Z*** is symmetric. **In fact, *P* does not influence the form of the cluster state, but only how much the nullifiers are squeezed**.

In fact, the covariance matrix $\{C\}_{j,k} = \langle\Psi|\frac{x_j\,x_k + x_k\,x_j}{2}|\Psi\rangle$ has to obey $\lim\limits_{z\to\infty} C = 0$

For example, if $\mathcal{P} = \mathbb{1} + e^{-i\Theta}\,\frac{\log\left(\mathcal{A}^2 + \mathbb{1}\right)}{2z}\,e^{i\Theta}$ ⇒ $C = e^{-2z}\,\mathbb{1}$

If instead $\mathcal{P} = \mathbb{1}$ ⇒ $C = \left(\mathcal{A}^2 + \mathbb{1}\right)\,e^{-2z}$

S. Zippilli, and D. Vitali , "*Possibility to generate any Gaussian cluster state by a multimode squeezing transformation* “; Phys. Rev. A 102, 052424 (2020); arXiv:2007.12772

Dissipative robust generation of CV cluster states with minimal resources

We consider the following **dissipative dynamics of N+1 bosonic modes, with only two ingredients**

$$\dot{\rho} = -\frac{\mathrm{i}}{\hbar}[H,\rho] + \mathcal{L}\rho \quad (1)$$

(a) $H = \hbar \sum_{j,k=0}^{N} \mathcal{J}_{j,k}\, b_j^\dagger\, b_k$ **Quadratic passive Hamiltonian**

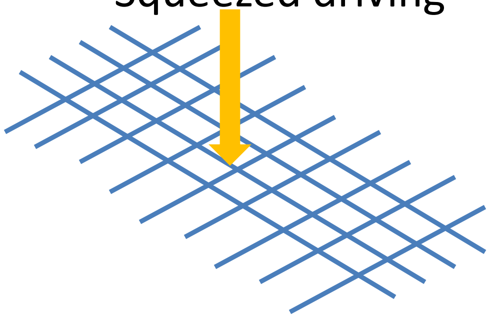
(b) $\mathcal{L}\rho = \kappa\{(\bar{n}+1)\,\mathcal{D}_{b_0,b_0^\dagger} + \bar{n}\,\mathcal{D}_{b_0^\dagger,b_0} - \bar{m}^*\,\mathcal{D}_{b_0,b_0} - \bar{m}\,\mathcal{D}_{b_0^\dagger,b_0^\dagger}\}\rho$

where $\mathcal{D}_{x,y}\,\rho = 2\,x\,\rho\,y - y\,x\,\rho - \rho\,y\,x$

$$|\bar{m}| = \sqrt{\bar{n}(\bar{n}+1)}$$

Single-site, pure squeezed reservoir acting on the "0" ancilla mode only, with coupling rate κ

Squeezed driving



The unique steady state of the above dynamics is $|\Psi_{tot}\rangle = |\psi_0\rangle\,|\Psi\rangle$

where $|\psi_0\rangle$ is a squeezed state of the ancilla mode, and **$|\Psi\rangle$ represents a wide class of pure Gaussian N-mode entangled states, including CV cluster states**

More precisely, the N-mode pure Gaussian entangled steady state **$|\Psi\rangle$** can generally be written as

$$|\Psi\rangle = U^{(p)}U^{(S)}|0\rangle \qquad U^{(S)} = \prod_{j=1}^{N} \exp\left[\frac{z_0}{2}\left(e^{i\varphi_j b_j^{+2}} - e^{-i\varphi_j b_j^2}\right)\right]$$

$$z_0 = \tanh^{-1}\sqrt{\bar{n}/(\bar{n}+1)}$$

Degree of squeezing of the reservoir

and ***U***$^{(p)}$ **passive unitary transformation, depending upon *H***

S. Zippilli, and D. Vitali , Phys. Rev. Lett. **126**, 020402 (2021); arXiv:2008.02539

This suggests an operational way to generate |Ψ> as steady state of the dissipative dynamics (1), with a properly engineered *H*

1. we choose a given **|Ψ>** (a cluster state with a given adjacency matrix ***A***)

2. we must "implement" ***U = U^(p) U^(S)*** : the squeezing transformation ***U^(S)*** is provided by the single-site squeezed reservoir

3. ***U^(p)*** is determined by ***H;*** many possible solutions for *H*; a simple one is given by a **linear chain with open boundaries,** transformed by a passive unitary depending upon ***A***

$$H^{(S)} = \mathrm{i}\hbar \sum_{j=0}^{N} J_j^{(S)} \left(e^{\mathrm{i}\theta_j} b_{j-1} b_j^\dagger - e^{\mathrm{i}\theta_j} b_{j-1}^\dagger b_j \right)$$

$$H = U_Z^{(p)} H^{(S)} U_Z^{(p)\dagger}$$

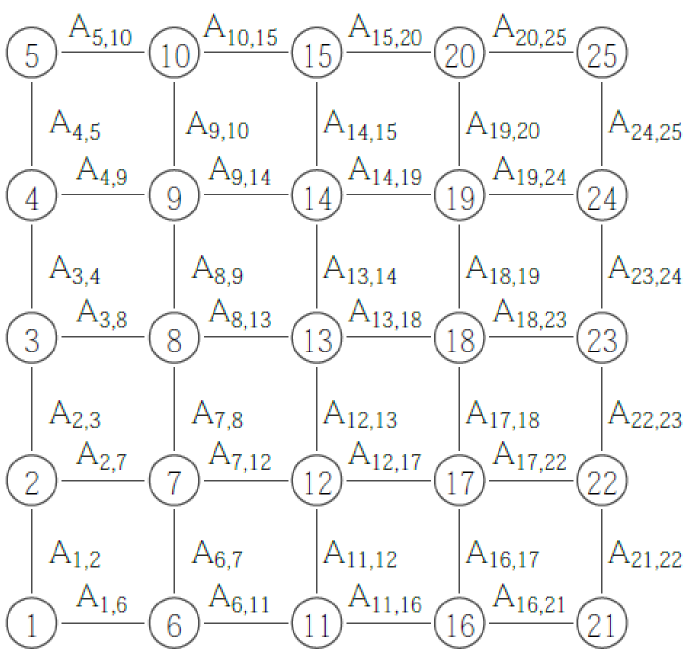
$$\theta_j = \frac{\varphi_j - \varphi_{j-1}}{2}$$

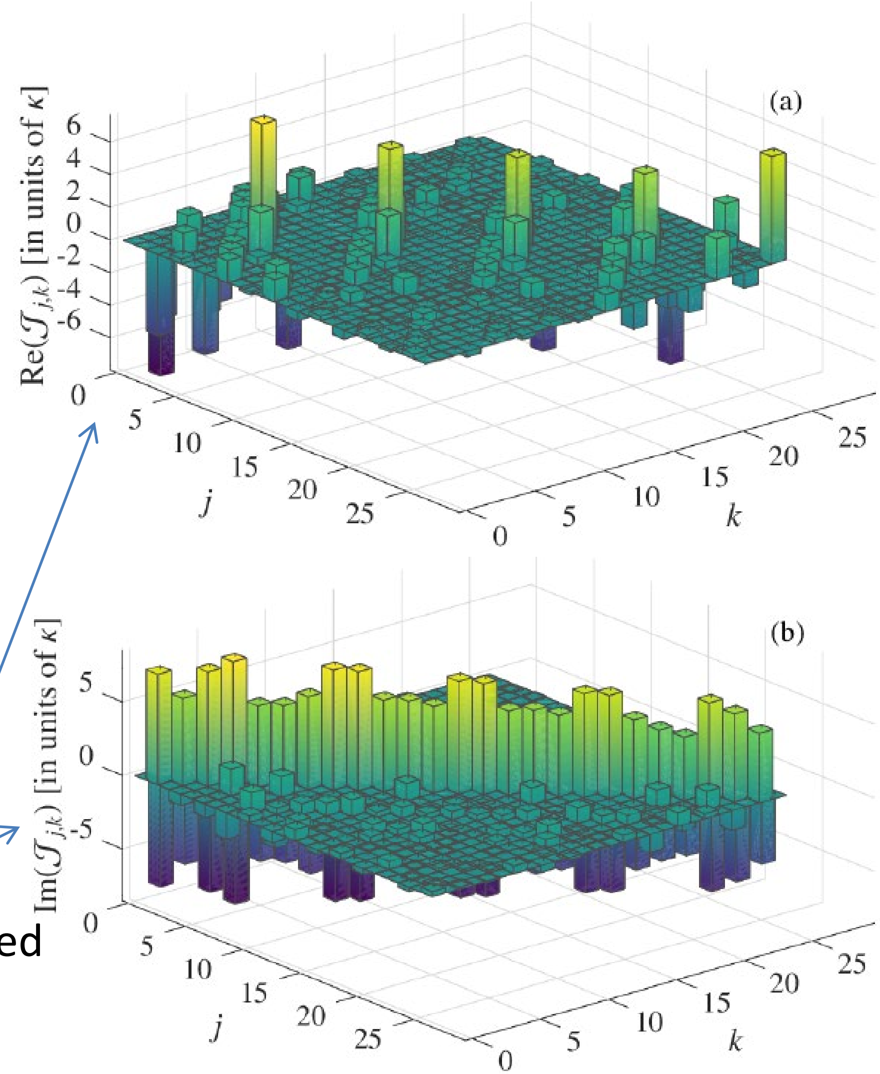
$$U_z^{(p)} b_j U_z^{(p)+} = \sum_{j=1}^{N} \left(\sqrt{\frac{i - \mathrm{A}}{i + \mathrm{A}}} \right)_{jk} b_k$$

Further freedom: the $J_j^{(S)}$ can be chosen freely. They affect only the rates at which the steady state is reached.

Example: N=25 square-lattice 2D cluster state (universal for MBQC)

The nonzero entries of ***A*** are all = 1 and are the links of the graph



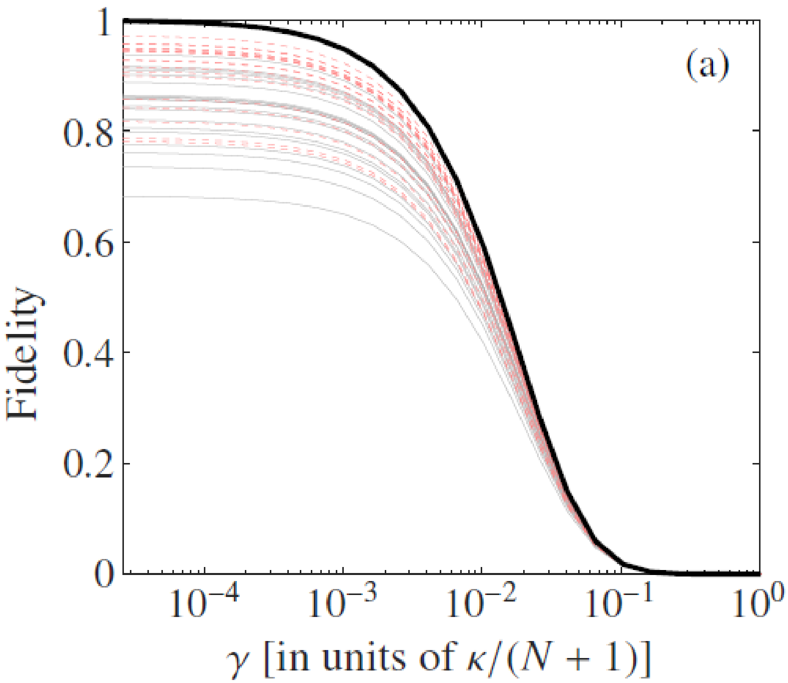


Coupling coefficients J_{jk} of the engineered passive bilinear Hamiltonian H

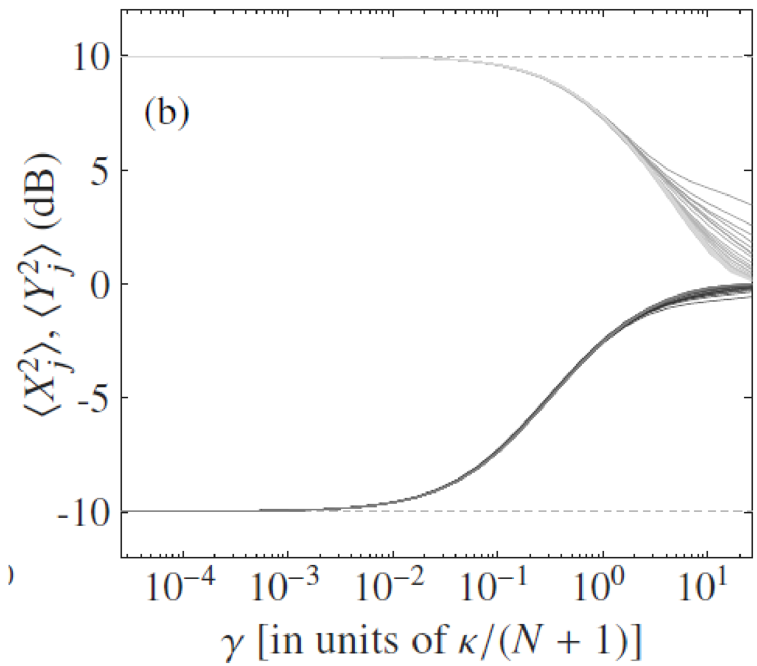
The generation is robust with respect to standard amplitude decay with rate γ

$$\dot{\rho}' = -\mathrm{i}\,[H,\rho'] + \mathcal{L}\rho' + \gamma \sum_{j=0}^{N} \mathcal{D}_{b_j,b_j^\dagger}\,\rho'$$

After optimising over the linear chain parameters $J_j^{(S)}$ for speeding up the convergence to the steady state



Fidelity wrt to the ideal pure state (thinner curves are for random errors of the parameters J_{kl} of the engineered H)



Variance of the squeezed nullifiers (and of the conjugated unsqueezed quadratures

Conclusions

- We provide a constructive method for generating an **arbitrarily chosen pure Gaussian cluster state of N bosonic modes** as unique stationary state of a dissipative dynamics

- Dynamics given by “cheap” resources: i) **a single-site squeezed reservoir;** ii) an **engineered passive bilinear Hamiltonian *H***

- The generation is robust wrt to: i) additional conventional dissipation; ii) random errors in the engineered Hamiltonian parameters

S. Zippilli, and D. Vitali , Phys. Rev. Lett. **126**, 020402 (2021); arXiv:2008.02539