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# BRIEF OVERVIEW OF BOSONIC QUANTUM SENSING PROTOCOLS

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# OUTLINE

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## General idea

### **Basic theoretical tools:**

Quantum channel discrimination (hypothesis testing)

Quantum channel estimation (metrology)

Channel simulation

Protocol stretching

Ultimate limits of discrimination/estimation

### **Some protocols:**

Quantum reading

Quantum illumination

Quantum radar

# QUANTUM SENSING – GENERAL IDEA

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Quantum sensing is a big umbrella name: quantum protocols able to outperform classical techniques in tasks of discrimination, estimation, resolution, etc.

Protocols are “quantum” because they beat “classical” benchmarks!

- Quantum illumination is “quantum” because it beats classical strategies (coherent states) in detecting a remote reflecting target (Guha, Shapiro, Lloyd, Zheshen, Wong et al.)
- Quantum reading is “quantum” because it beats classical strategies in retrieving classical information from an optical digital memory
- Quantum parameter estimation is “quantum” because it beats the SQL (Datta, Braunstein)
- Quantum optical resolution is “quantum” because it beats the Rayleigh diffraction limit (M. Tsang)
- Entangled-sensor networks are “quantum” because they beat sensor Networks based on separable states, e.g., for SL (Zhuang, Zhang)

Therefore it is important:

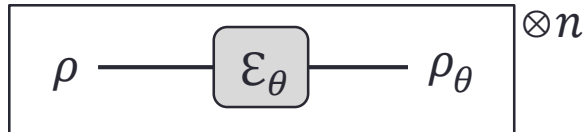
- 1) A clear identification of the **classical benchmark**
- 2) A proof that we can do better with **quantum setups**

A related problem is to explore their ultimate limits

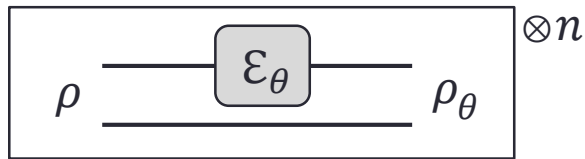
# TYPES OF PROTOCOLS

## Block protocols

### (b) Block unassisted

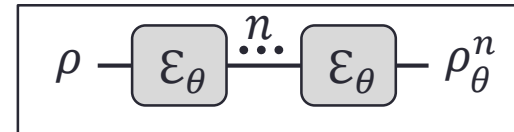


### (c) Block assisted

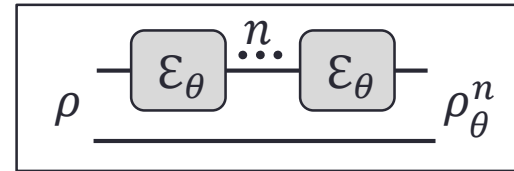


## Sequential protocols

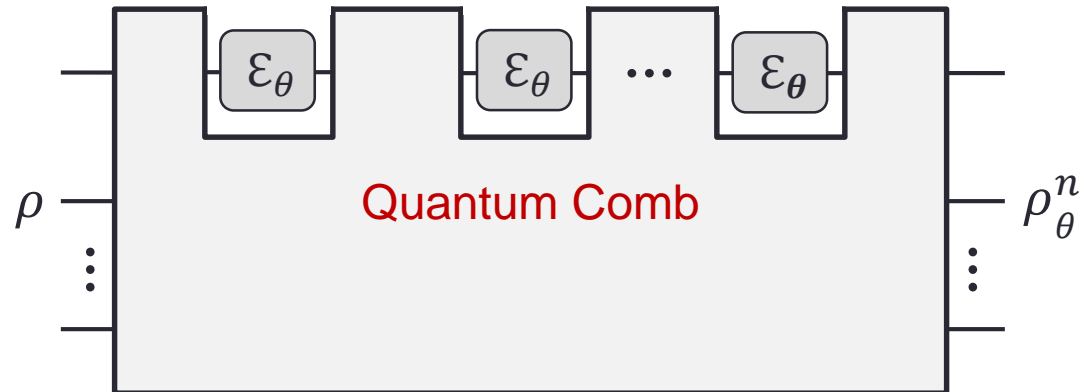
### (d) Sequential unassisted



### (e) Sequential assisted



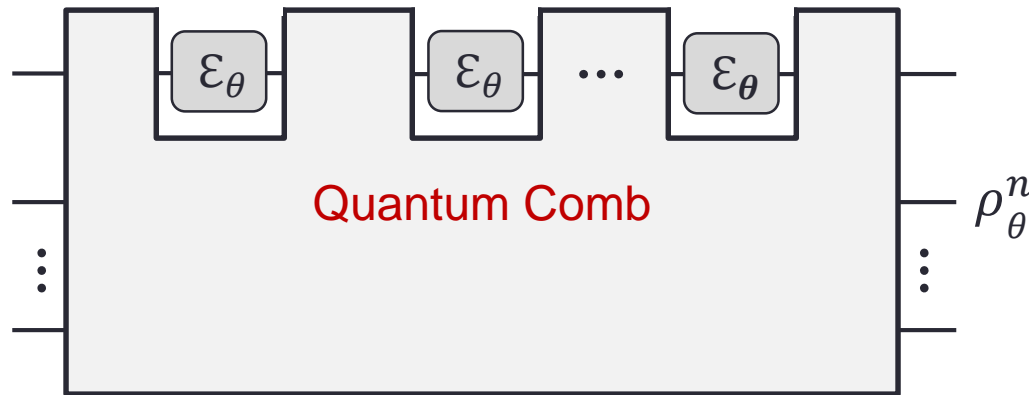
## Adaptive protocols



Pirandola, Bardhan, Gehring, Weedbrook, Lloyd

*Advances in Photonic Quantum Sensing*, Nature Photonics 12, 724-733 (2018)

# CHANNEL DISCRIMINATION AND ESTIMATION



## Discrimination:

- Parameter  $\theta$  takes two values  $\{\theta_0, \theta_1\}$  with the same probability
- Binary discrimination between  $\mathcal{E}_0 = \mathcal{E}_{\theta_0}$  and  $\mathcal{E}_1 = \mathcal{E}_{\theta_1}$
- Error probability for symmetric discrimination:

Helstrom bound, quantum Chernoff bound, fidelity

(False-negative probability in asymmetric discrimination

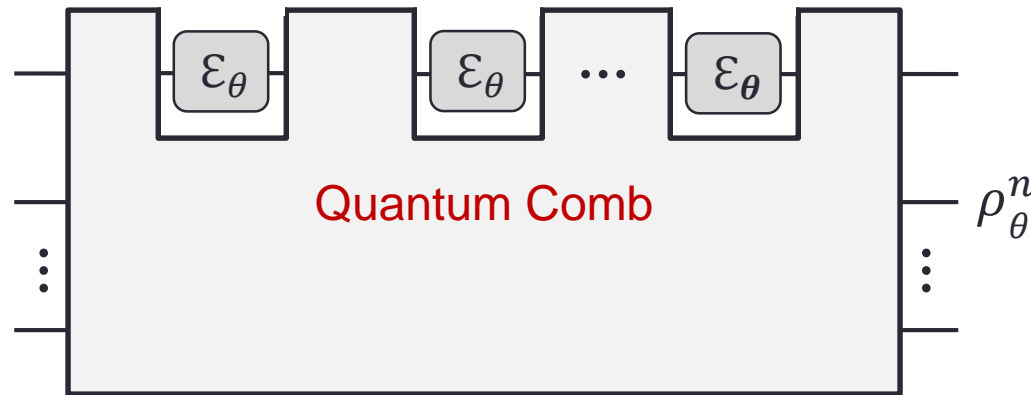
Relative entropy, quantum Hoeffding bound)

- Problem is the optimization over the combs

$$p_{\text{err}} = \frac{[1 - D(\rho_0^n, \rho_1^n)]}{2}$$

$$p_{\text{err}}(\mathcal{E}_0, \mathcal{E}_1) = \max_{\text{Combs}} p_{\text{err}}$$

# CHANNEL DISCRIMINATION AND ESTIMATION



## Estimation:

- Parameter  $\theta$  takes continuous values
- Construct unbiased estimator  $\tilde{\theta}$  with error variance  $\delta\theta^2$
- Quantum Cramer Rao bound (QCRB)
- QFI = quantum Fisher information

$$\text{QFI}(\rho_\theta^n) = \frac{8[1 - F(\rho_\theta^n, \rho_{\theta+d\theta}^n)]}{d\theta^2}$$

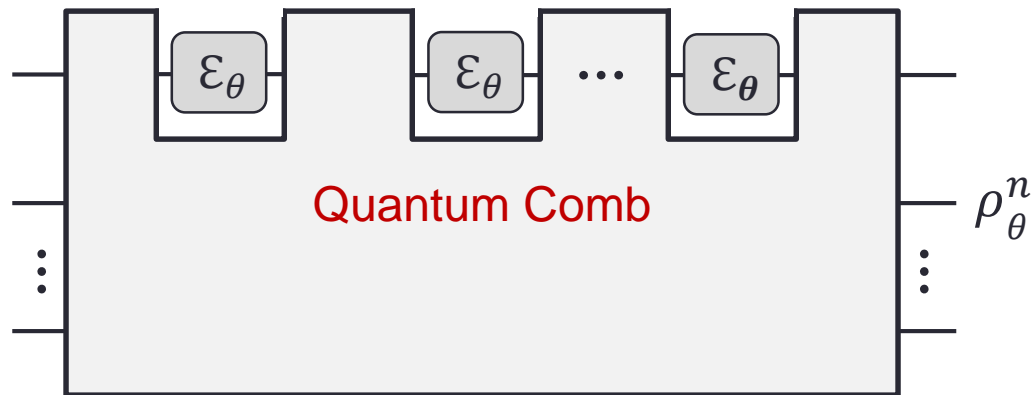
$$\delta\theta^2 \geq \frac{1}{\text{QFI}(\rho_\theta^n)}$$

➤ Problem is the optimization over the combs

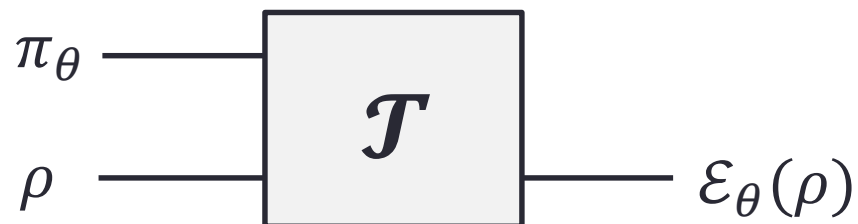
$$\overline{\text{QFI}} = \max_{\text{Combs}} \text{QFI}$$

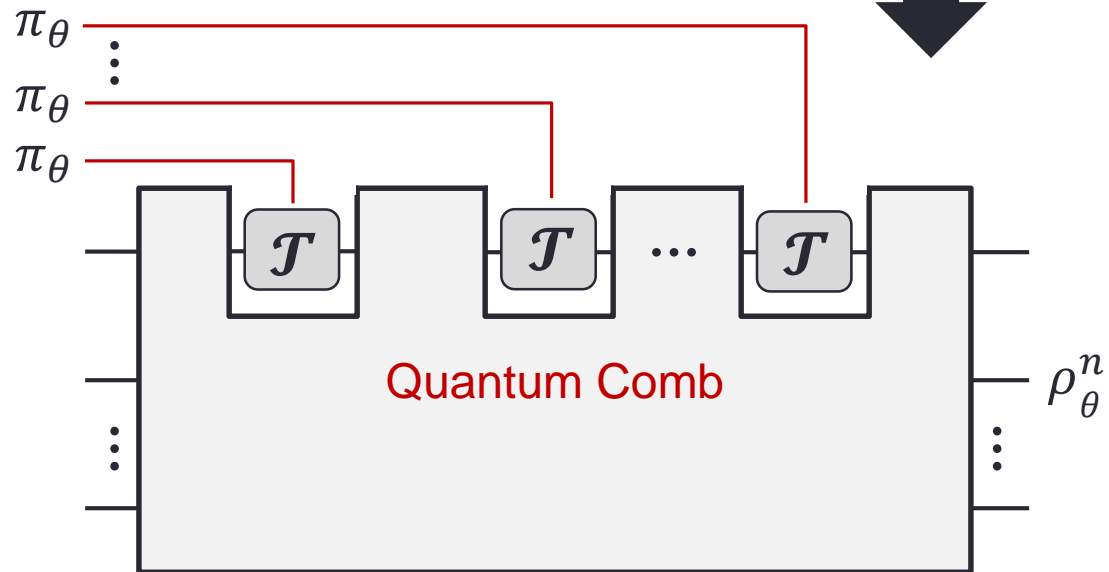
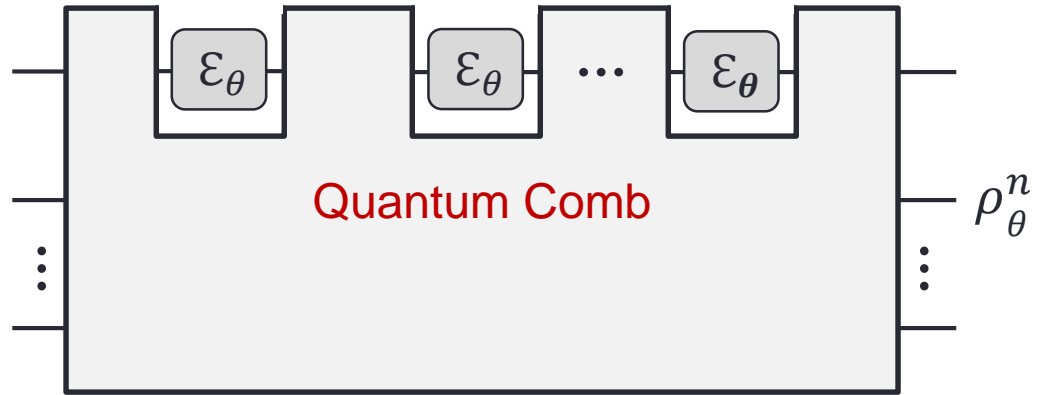
# CHANNEL SIMULATION

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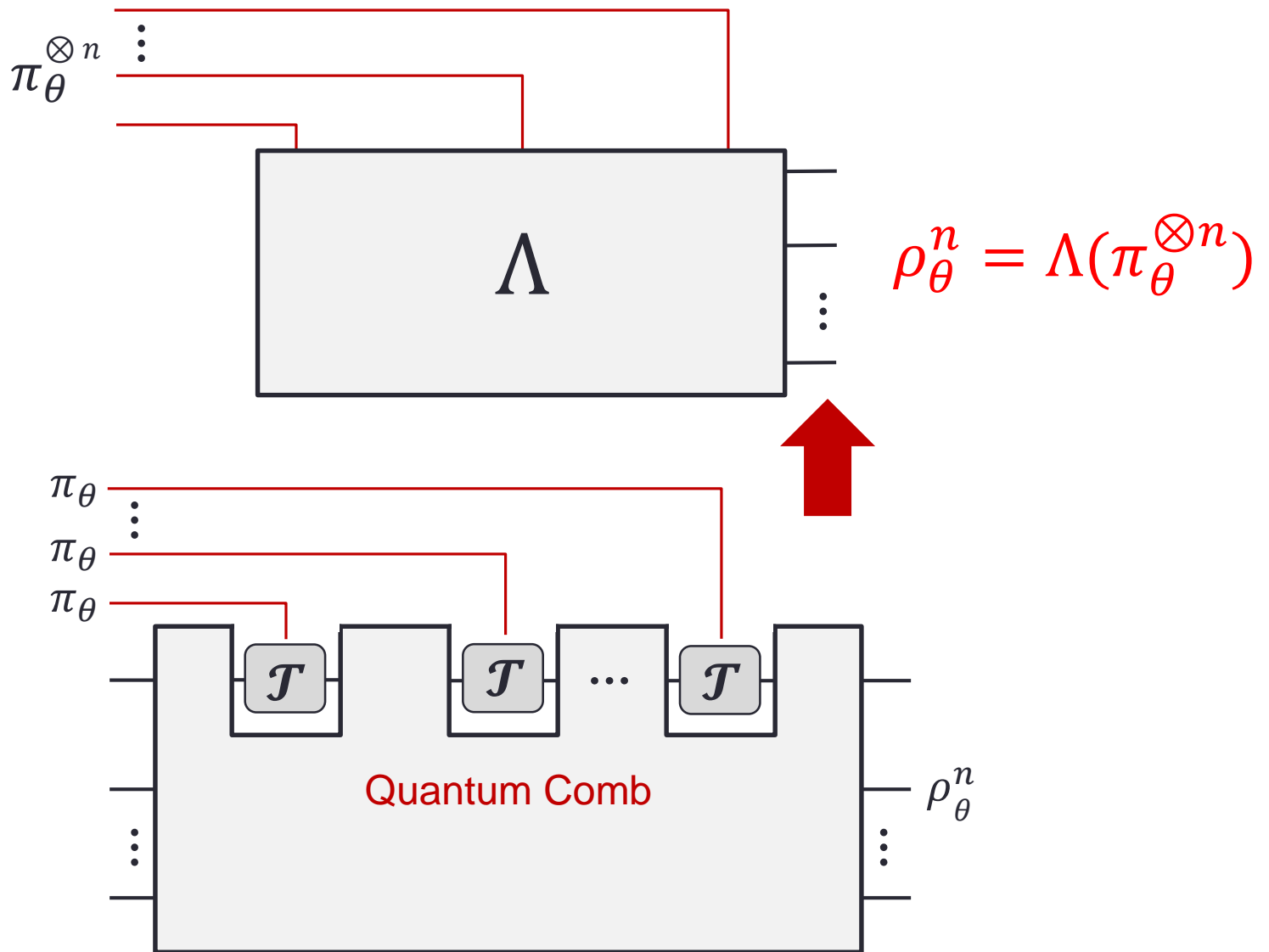


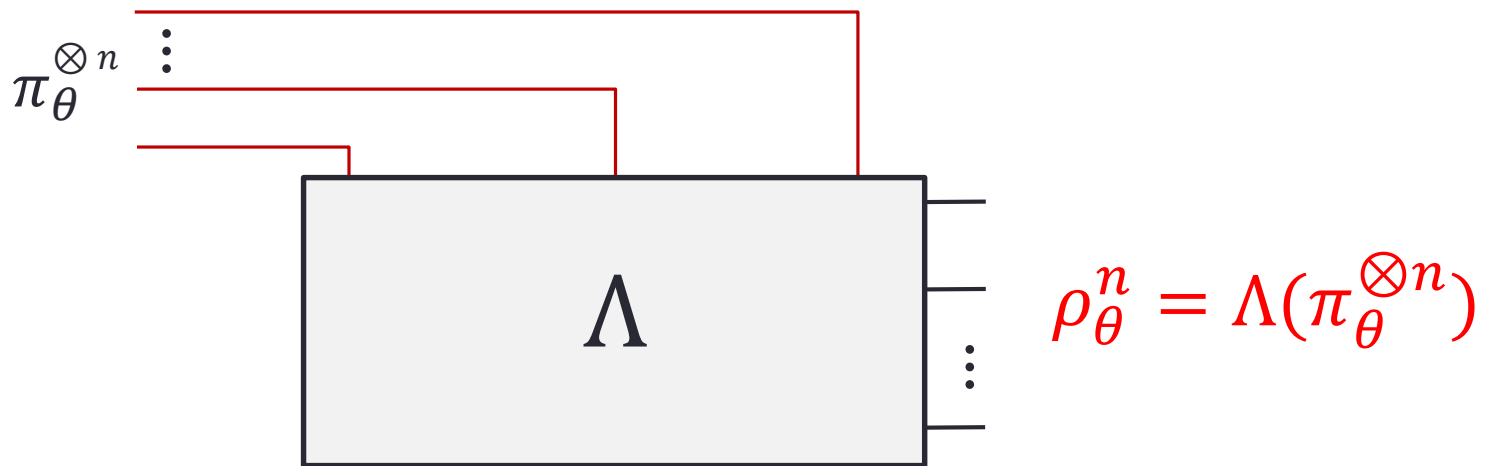
Suppose that the unknown channel is **jointly-simulable** with some trace-preserving quantum operation and some program state











## Adaptive bounds

Channel discrimination:

$$p_{\text{err}}(\mathcal{E}_0, \mathcal{E}_1) \geq [1 - D(\pi_{\theta_1}^{\otimes n}, \pi_{\theta_2}^{\otimes n})]/2,$$

Parameter estimation:

$$\overline{\text{QFI}} = \max_{\text{Combs}} \text{QFI} \leq \text{QFI}(\pi_{\theta})$$

# ULTIMATE LIMITS OF QUANTUM DISCRIMINATION & ESTIMATION

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Known results for jointly teleportation-covariant channels.

Adaptive discrimination/estimation of:

- Pauli channels (depolarizing, dephasing)
- Erasure channels
- Thermal or additive noise in bosonic Gaussian channels

Open problems for the other channels.

Adaptive discrimination/estimation of:

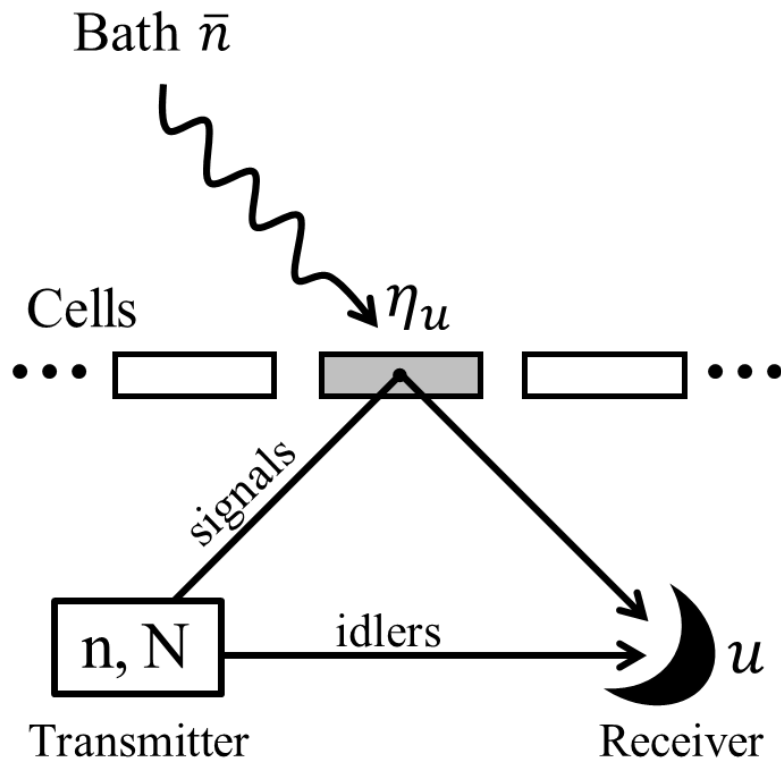
- Amplitude damping channels
- **Bosonic loss**



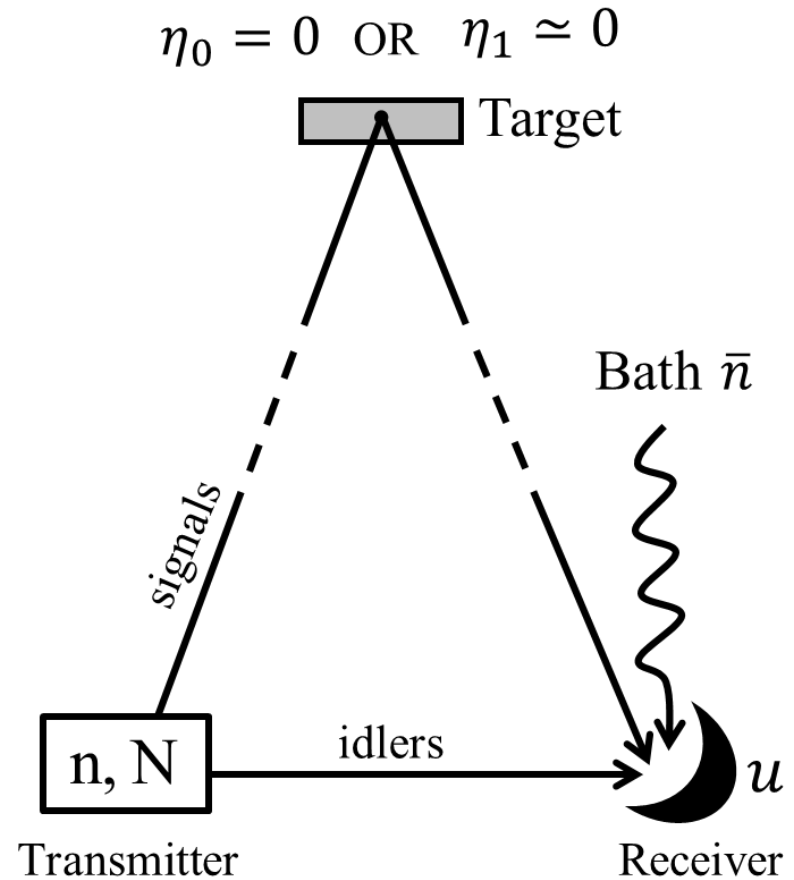
It is not yet known the best performance of protocol of quantum sensing which are based on the symmetric discrimination or estimation of bosonic loss. Including the protocols of quantum reading and illumination.

# QUANTUM SENSING PROTOCOLS

## Quantum Reading



## Quantum Illumination

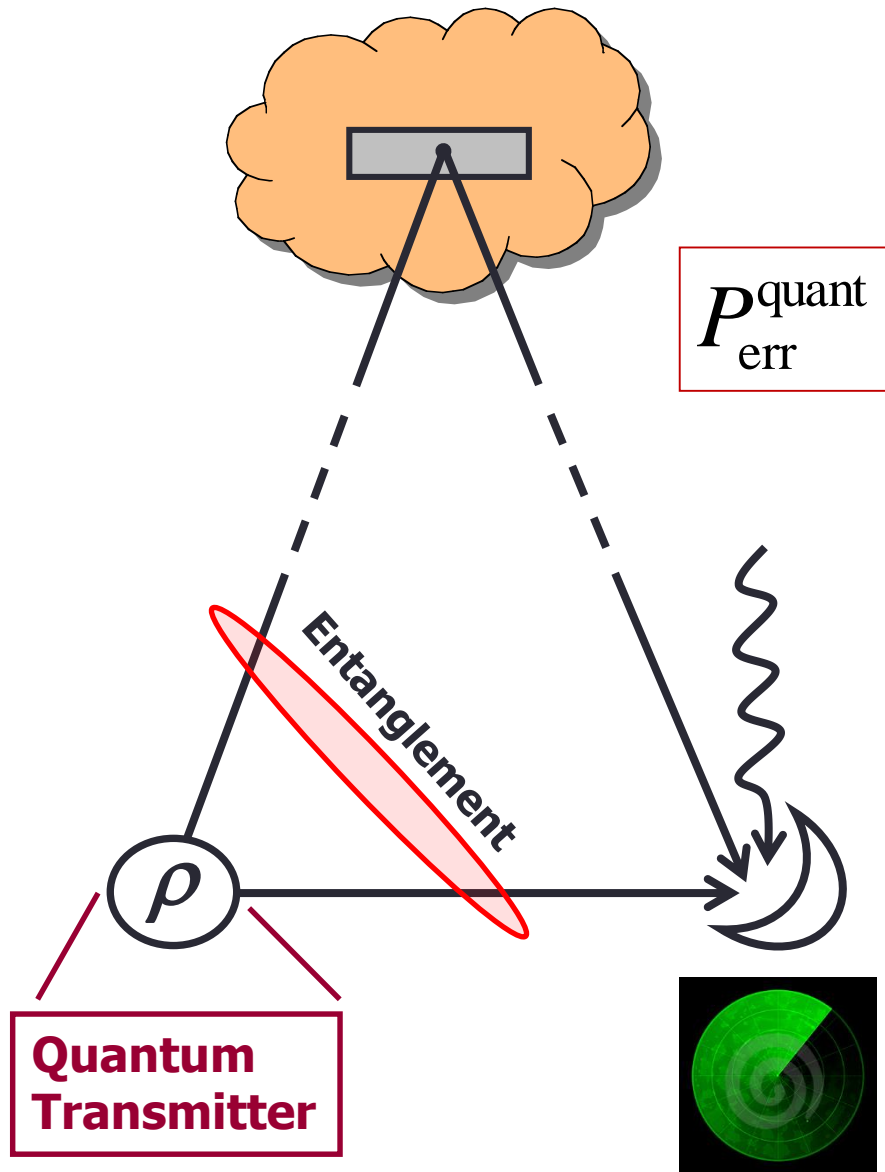


Ultimate limits are still unknown

# QUANTUM ILLUMINATION

Tan et al. PRL 101, 253601 (2008)

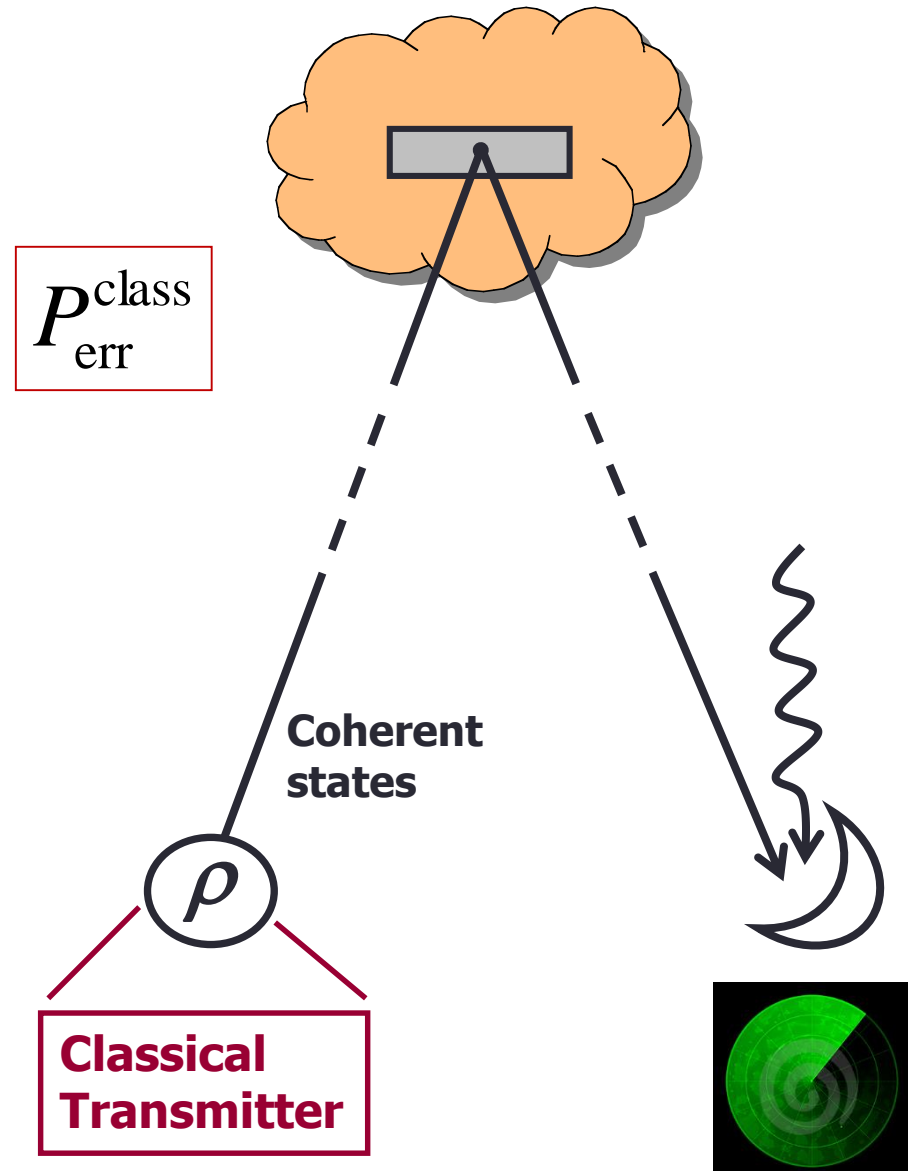
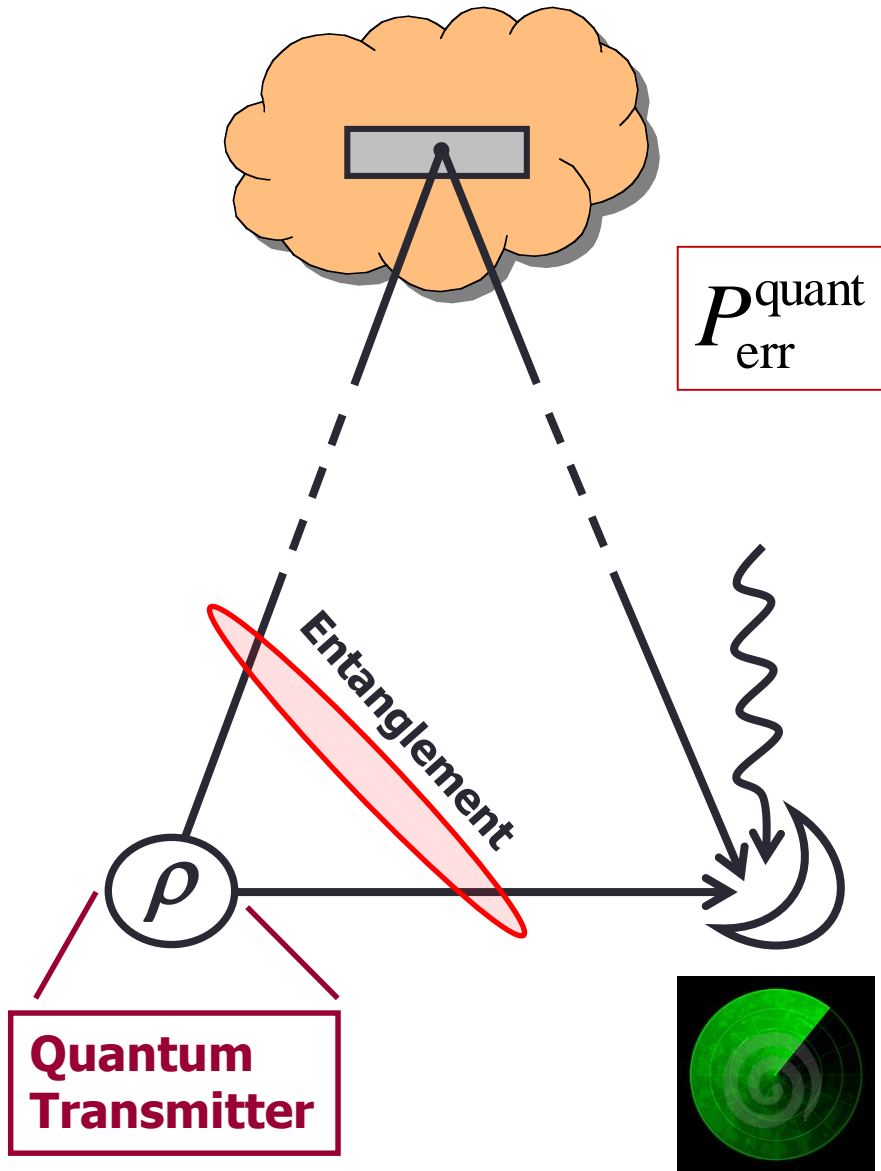
Zhuang et al. PRL 118, 040801 (2017).



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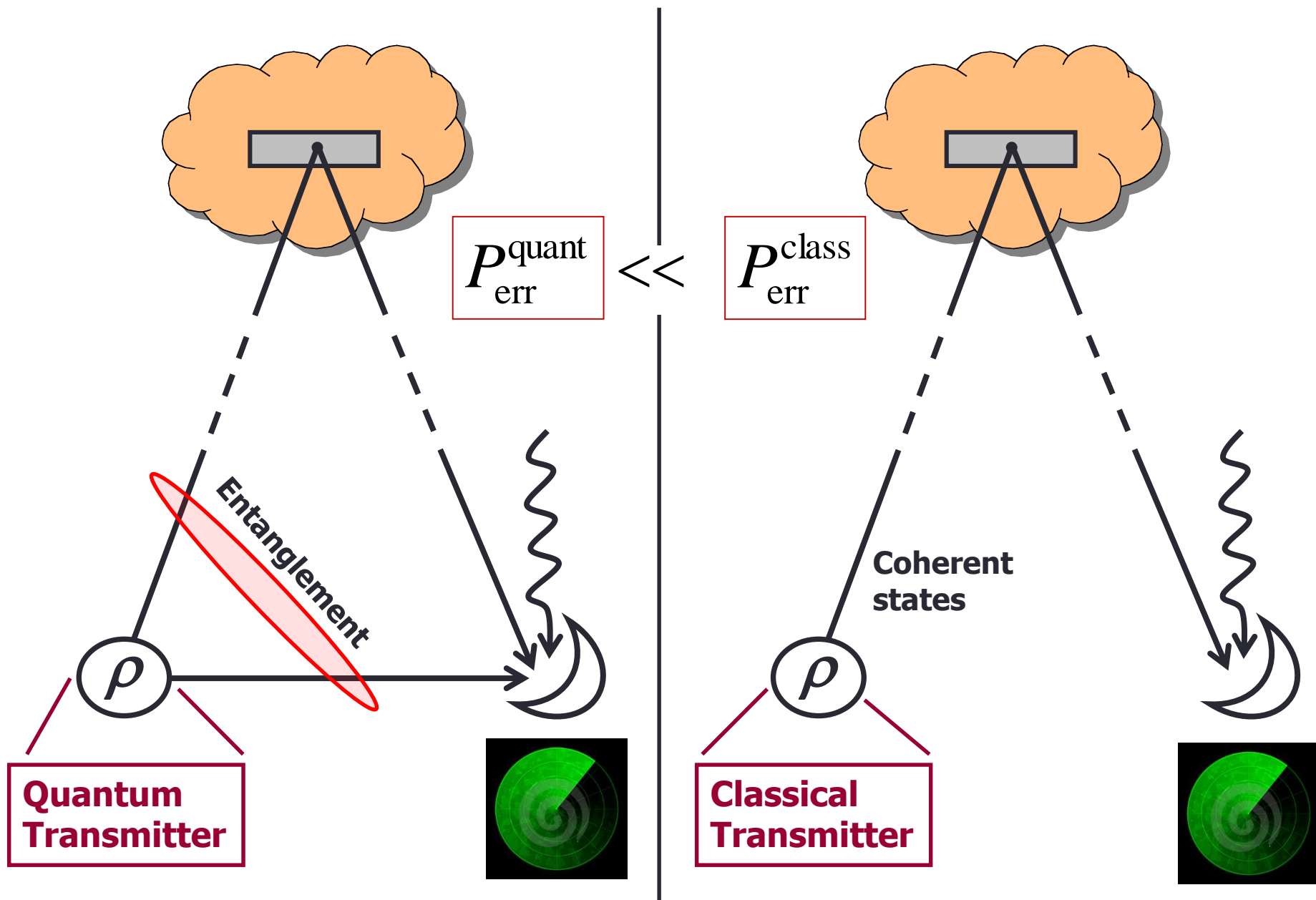
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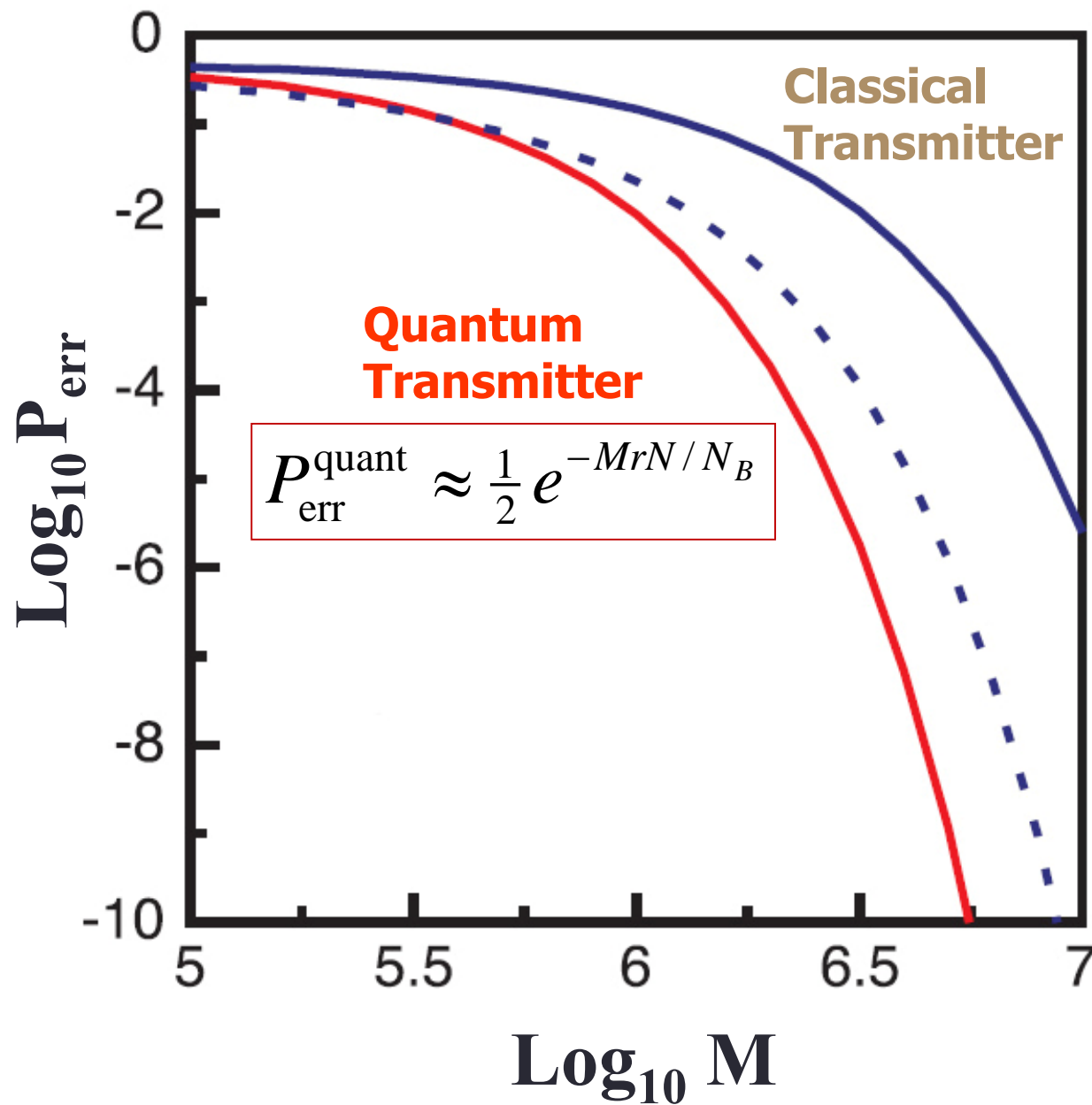


# QUANTUM ILLUMINATION

Tan et al. PRL 101, 253601 (2008)  
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# QUANTUM ILLUMINATION



$$P_{\text{err}}^{\text{class}} \approx \frac{1}{2} e^{-MrN/4N_B}$$

$$P_{\text{err}}^{\text{quant}} \approx \frac{1}{2} e^{-MrN/N_B}$$

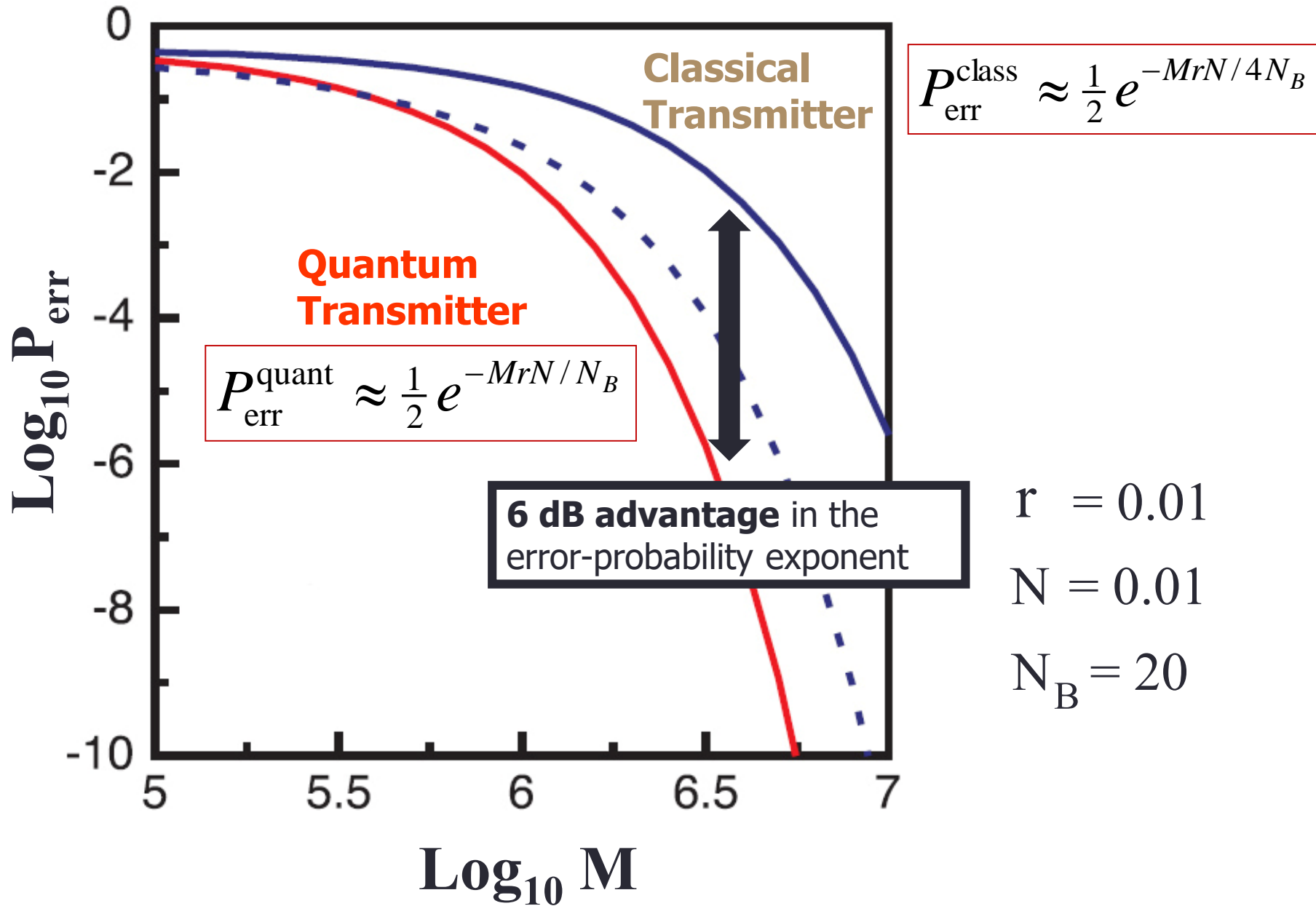
$$r = 0.01$$

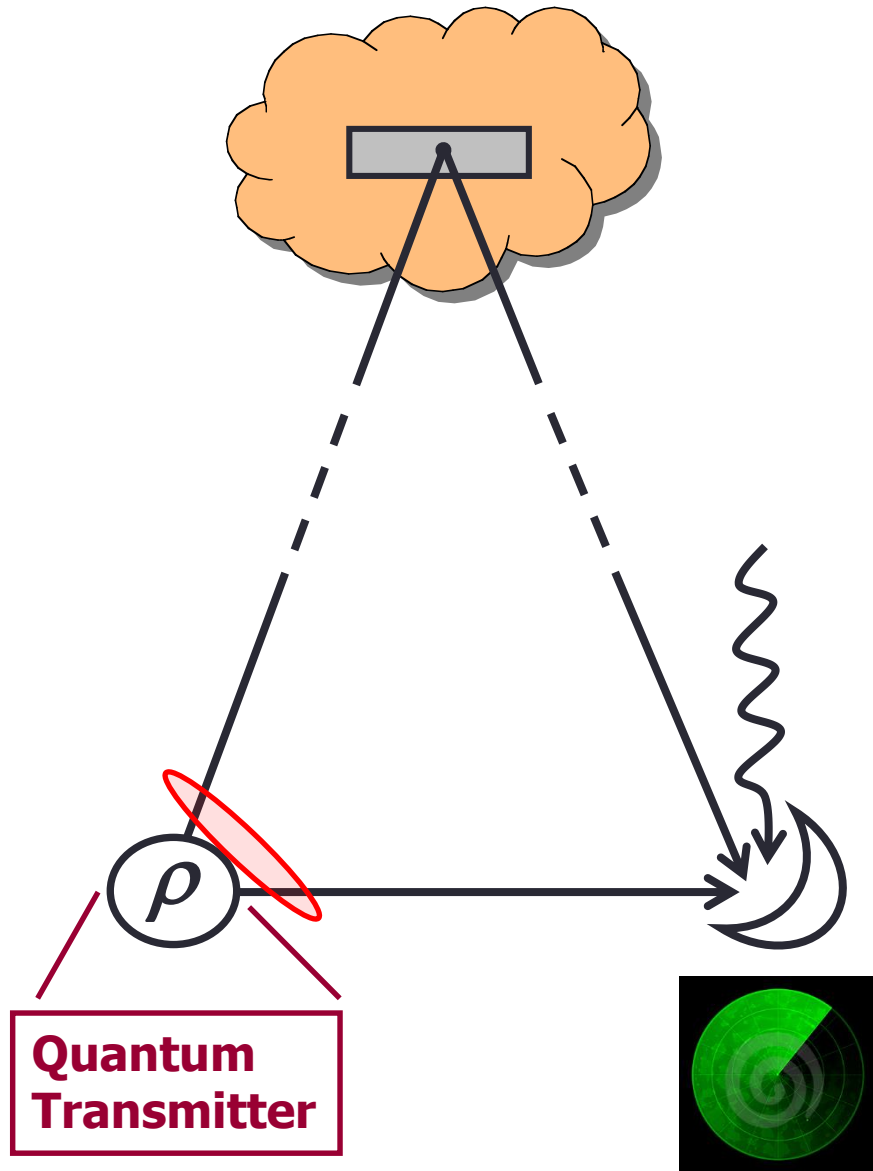
$$N = 0.01$$

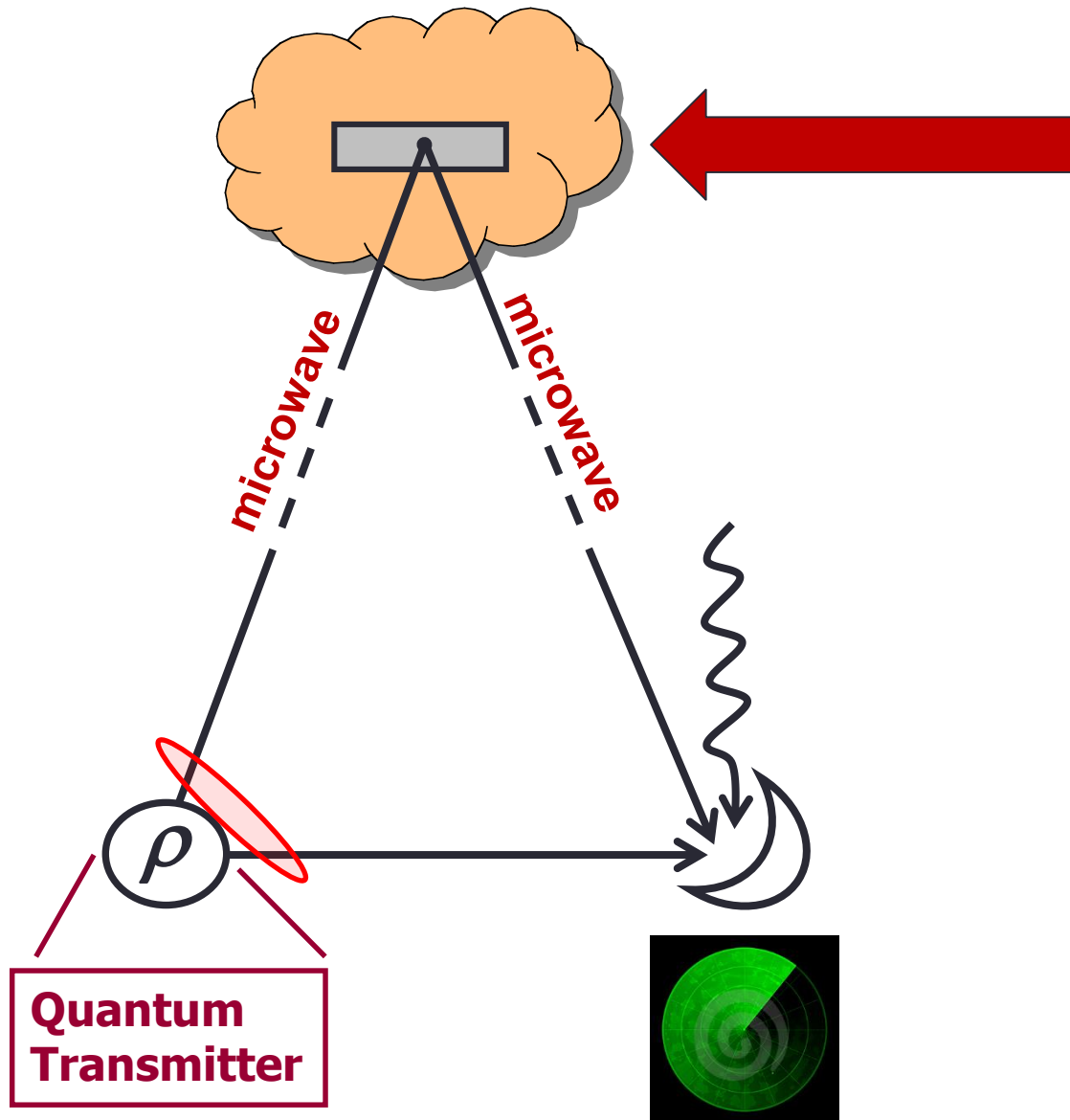
$$N_B = 20$$



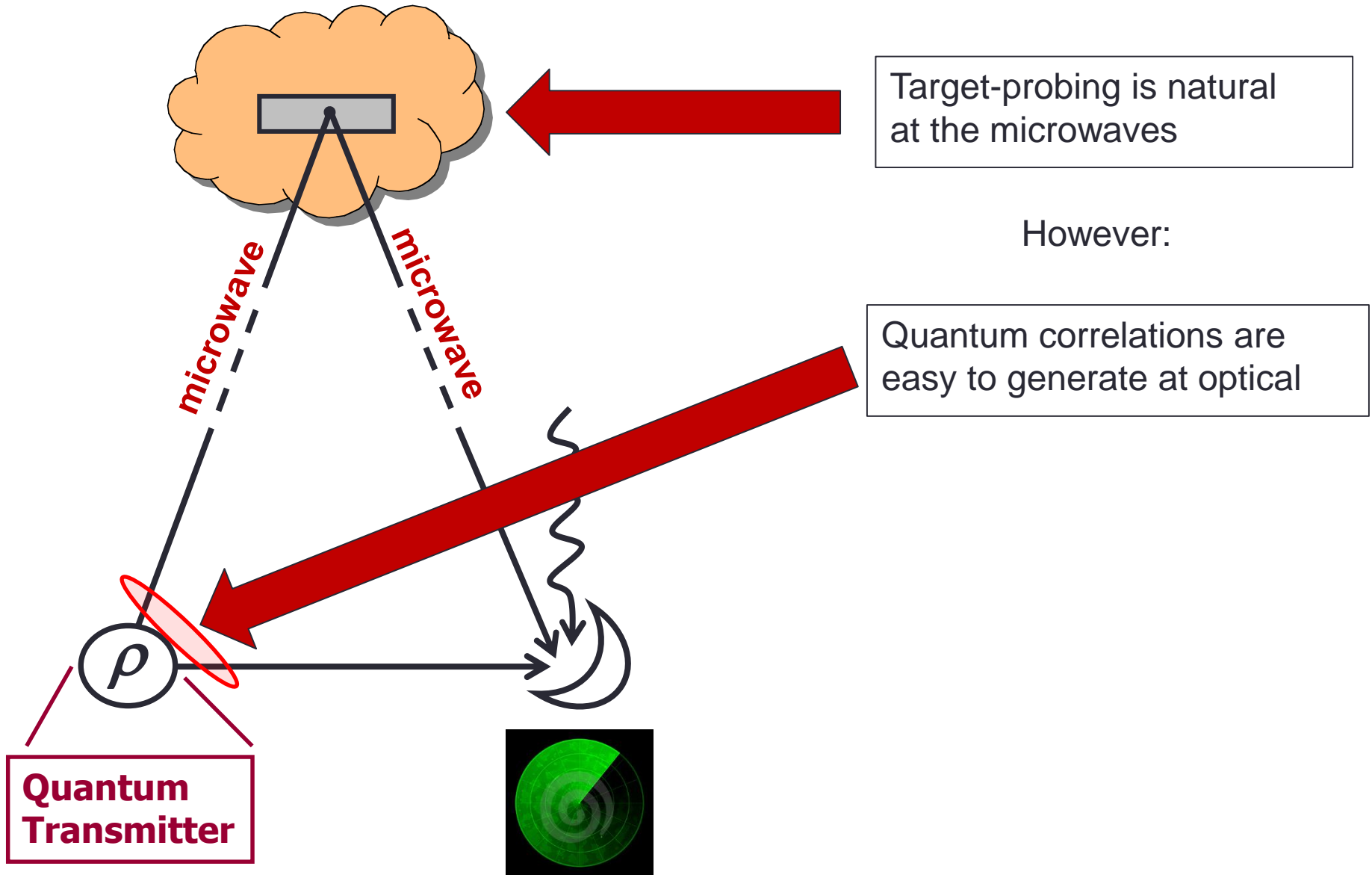
# QUANTUM ILLUMINATION

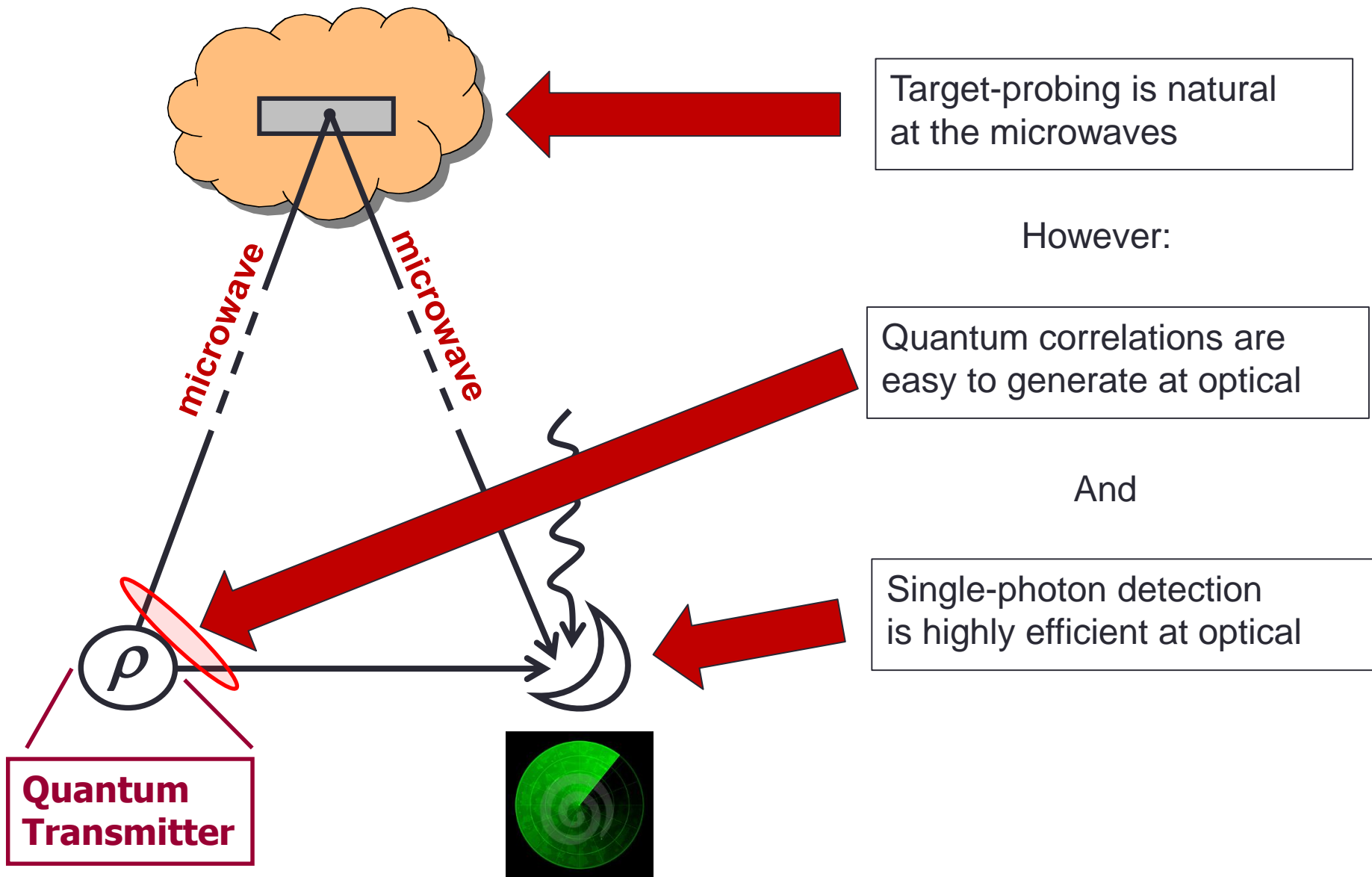


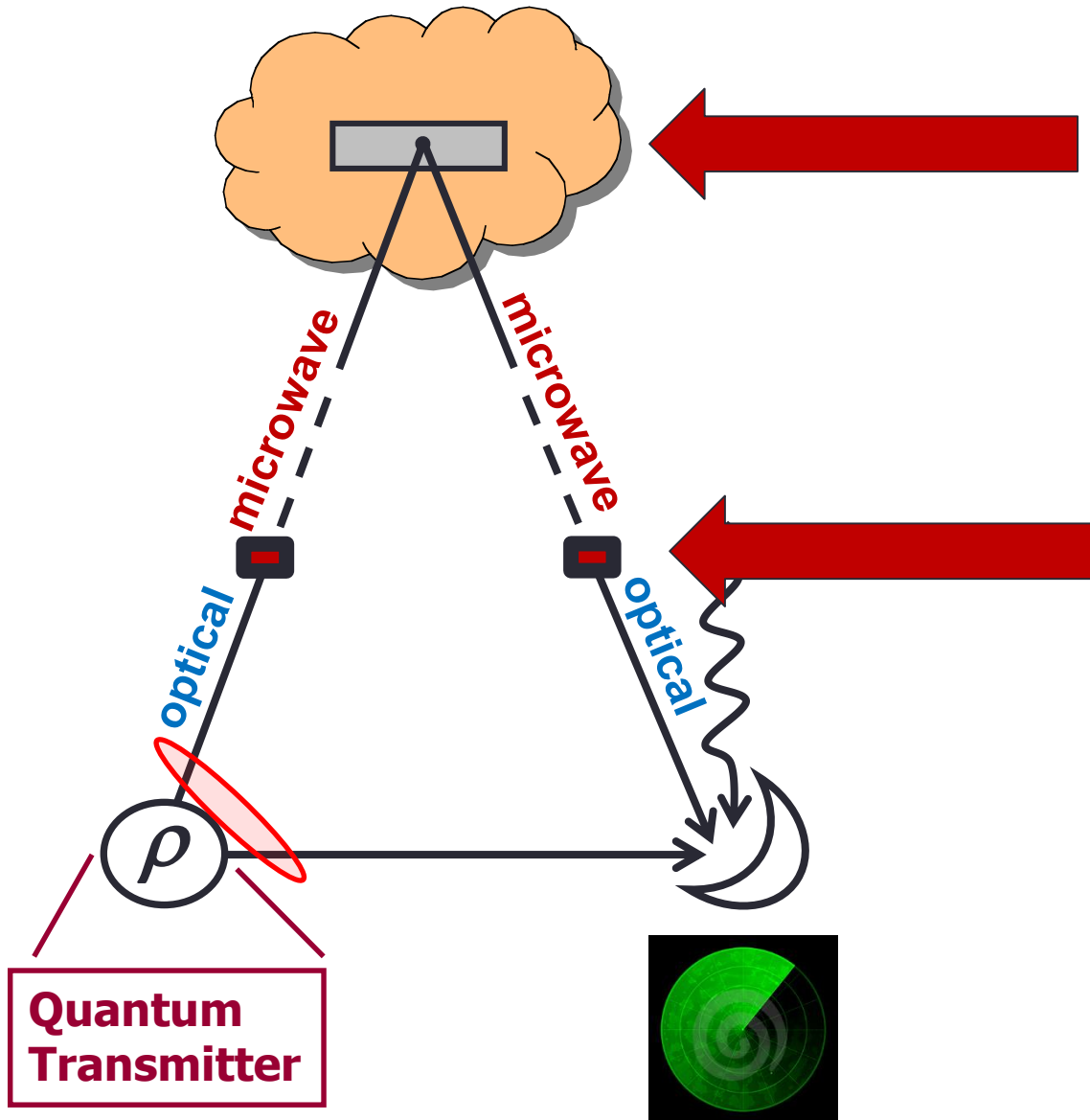




Target-probing is natural  
at the microwaves

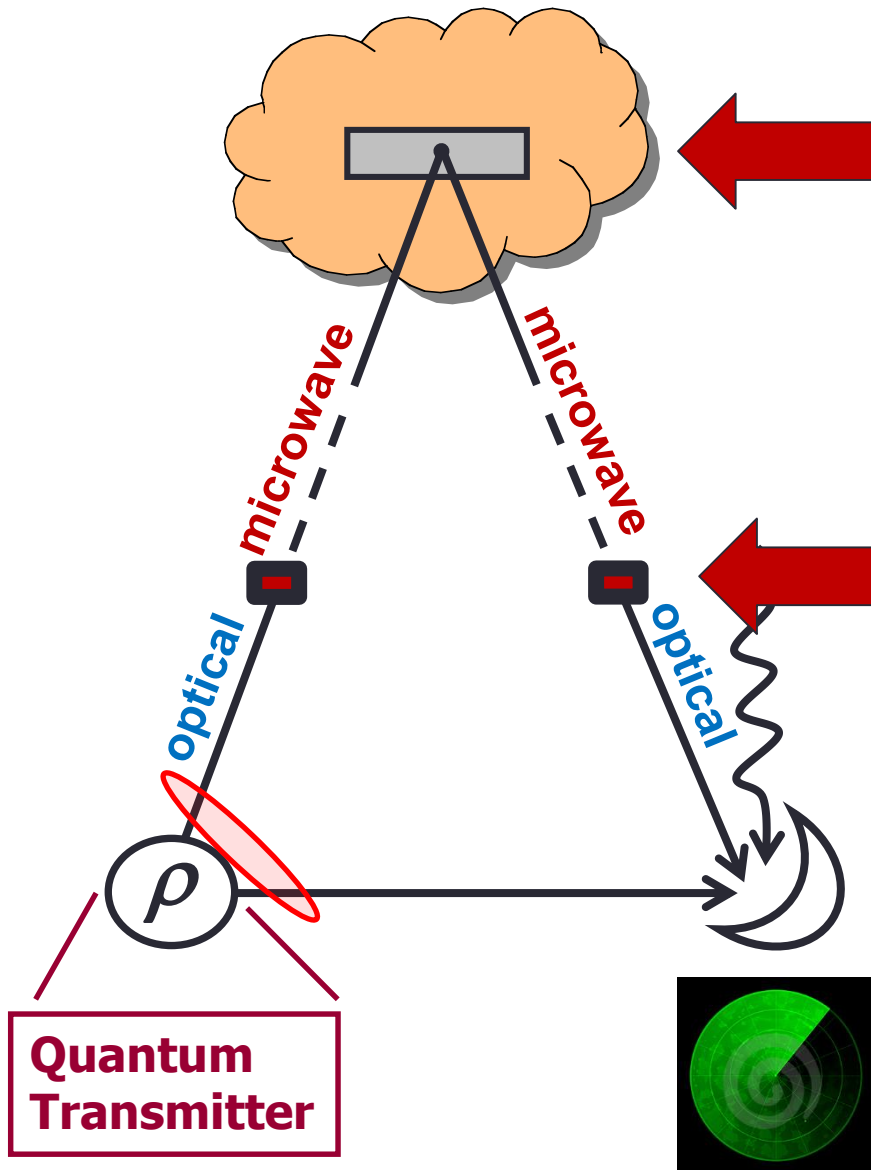






Target-probing is natural at the microwaves

Possible solution:  
**Optical-microwave** transducers



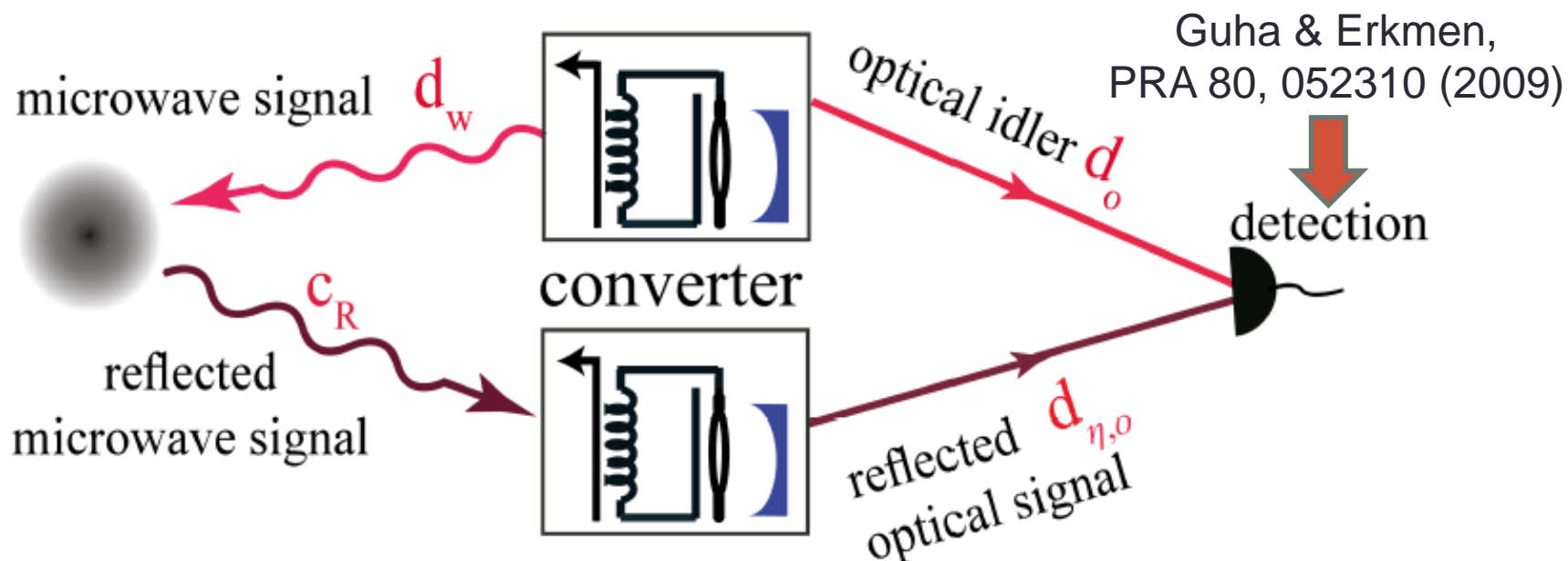
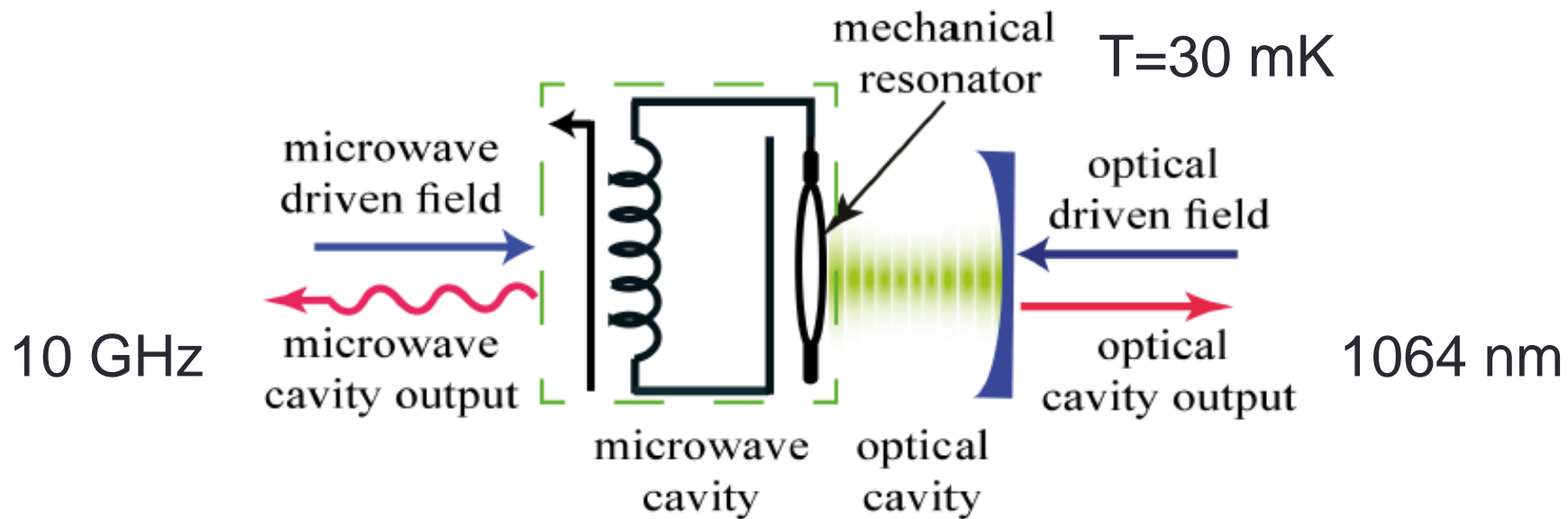
Target-probing is natural at the microwaves

Possible solution: **Optical-microwave transducers**

Needs good I/O efficiency in converting entanglement and quantum coherence

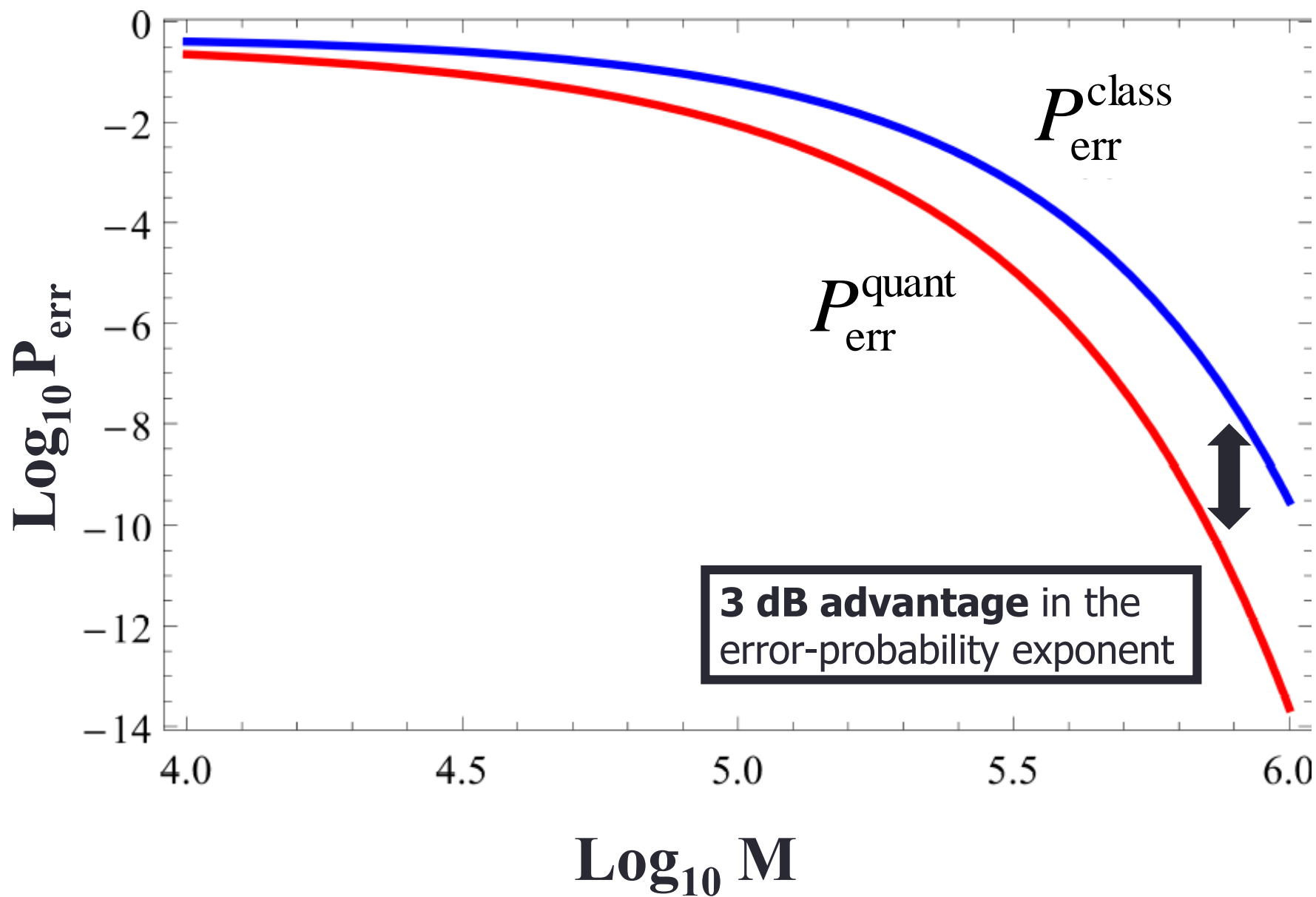
Theory: efficiency 100%  
Exp: efficiency 74%\*

\*Reiserer, Ritter, Rempe, Science 342, 1349 (2013)



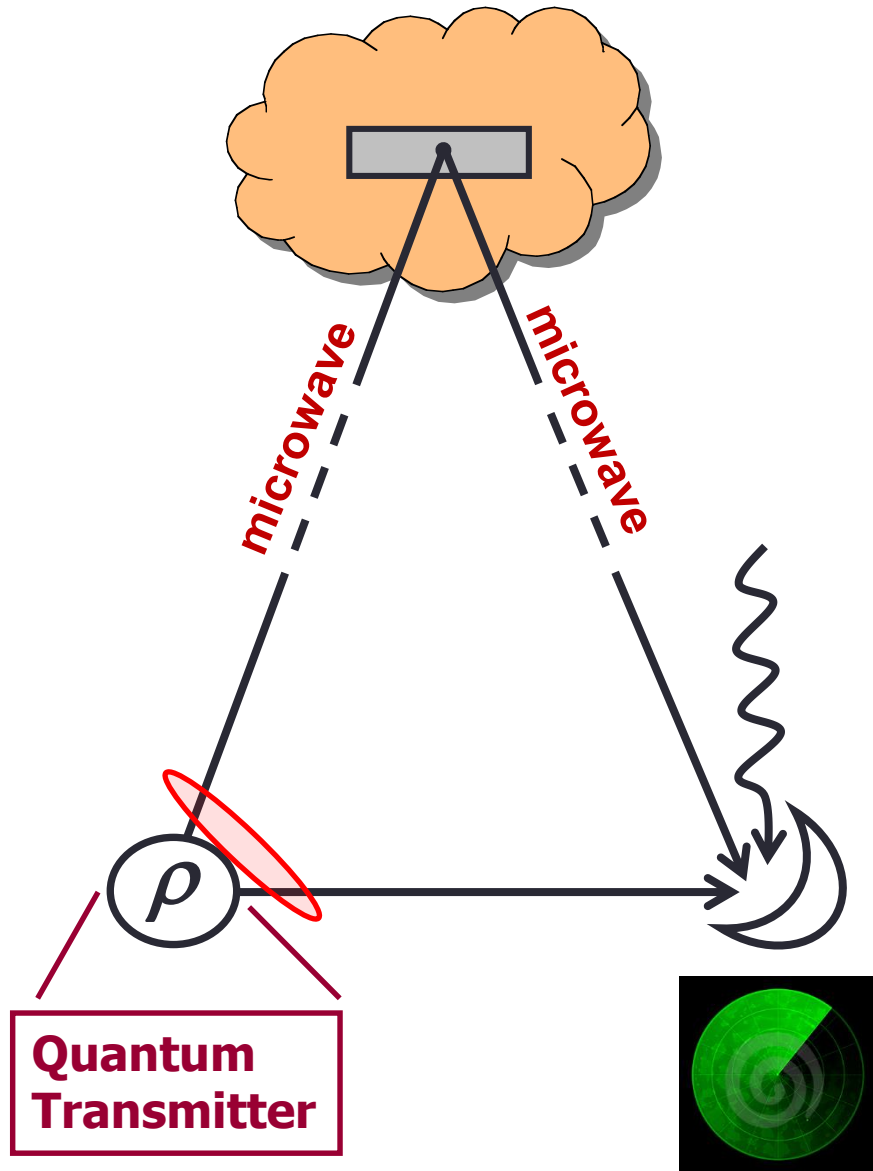


# MICROWAVE QUANTUM ILLUMINATION



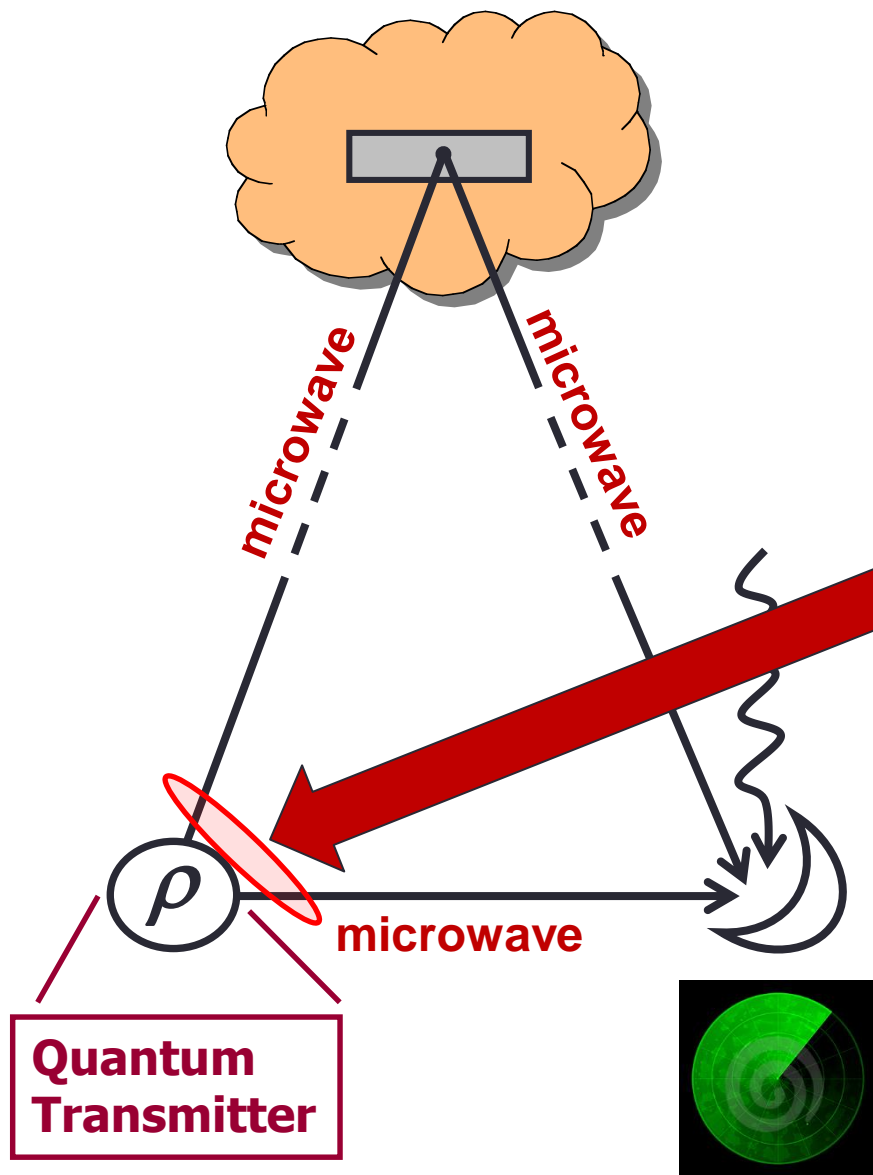
# MICROWAVE QUANTUM ILLUMINATION

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Other quantum technologies?

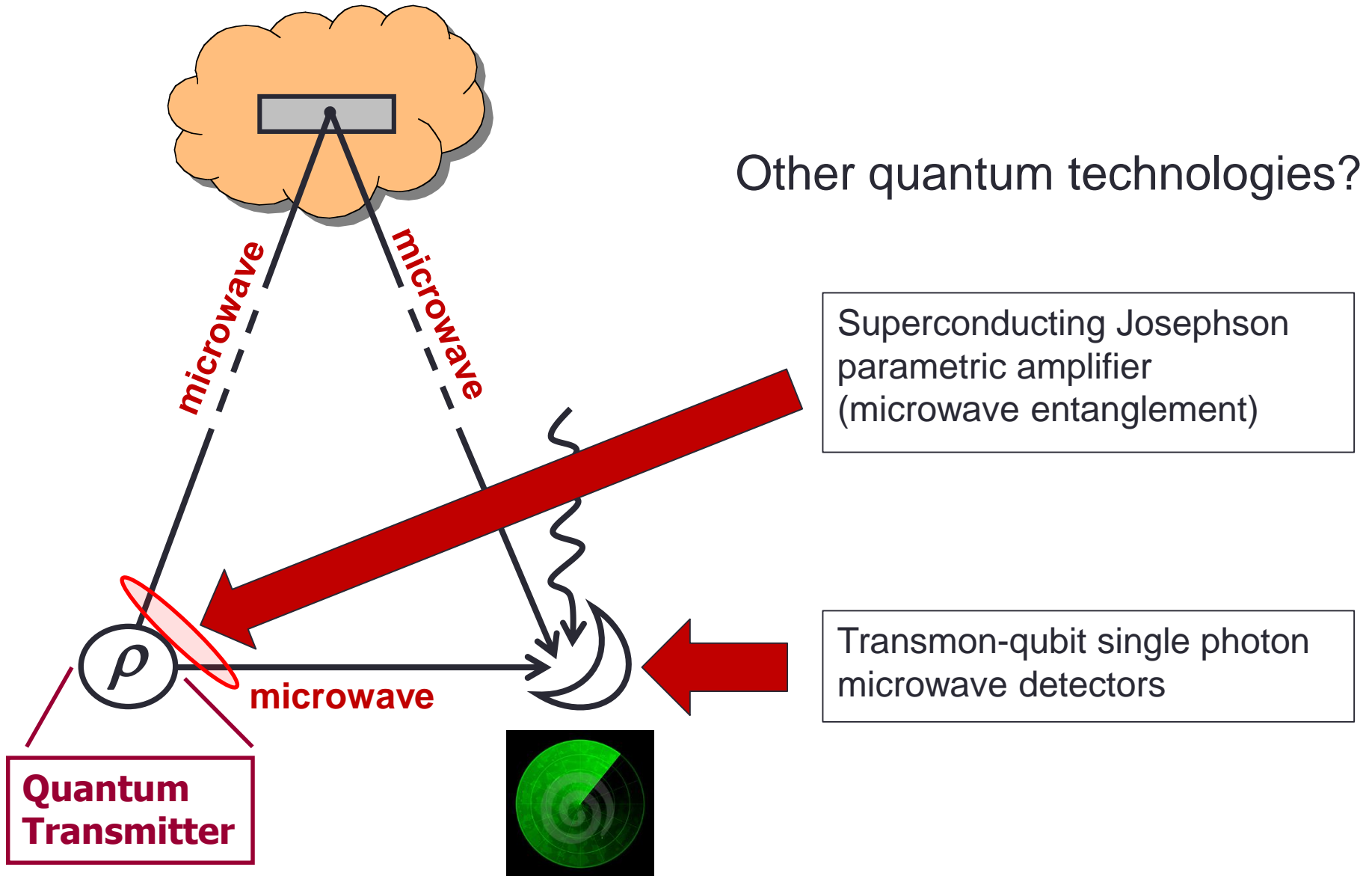
# MICROWAVE QUANTUM ILLUMINATION



Other quantum technologies?

Superconducting Josephson  
parametric amplifier  
(microwave entanglement)

# MICROWAVE QUANTUM ILLUMINATION



# QUANTUM RADAR

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- Confusion between LIDAR (optical) and RADAR (microwave)
- Quantum illumination is not the only possible design
- People confused about the range
- Much hype in terms of defence applications
- Definition of “quantum versus classical” is different between quantum information community and radar community

Our benchmark = beating the error probability of coherent states  
Their benchmark = SNR from the Albersheim's equation

Our usual definition of quantum radar has an entangled source

More general definition of quantum radar: **setup with a quantum part**  
(e.g., it may be only detection)

# CLASSICAL vs QUANTUM RADAR

## Classical radar

Radar equation

$$P_r = \frac{P_t G_t A_r \sigma F^4}{(4\pi)^2 R^4}$$

Background noise

$$P_n = k_B T B_n F_n$$

$$\text{SNR} = \frac{P_r}{P_n} \simeq 10 \div 20\text{dB}$$

Albersheim's equation



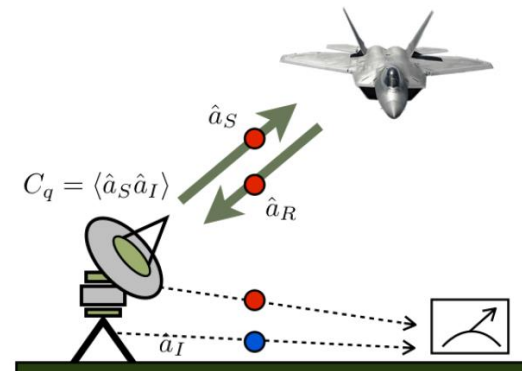
## Quantum (entangled) radar

Quantum Chernoff bound

Quantum relative entropy

Quantum Hoeffding bound

$$\text{SNR} = \frac{rN}{N_B} \frac{\text{Received photons}}{\text{Background photons}}$$
$$\simeq -100 \div -10\text{dB}$$



# CLASSICAL vs QUANTUM RADAR

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## Classical radar

High energetic

~ MW per pulse

Large attenuations

~  $10^{-20}$

Long range

~ 30 ÷ 100 km

## Quantum (entangled) radar

Low energetic

< 1 ph per mode

Large background noise

100 ph per mode

Presumably short range

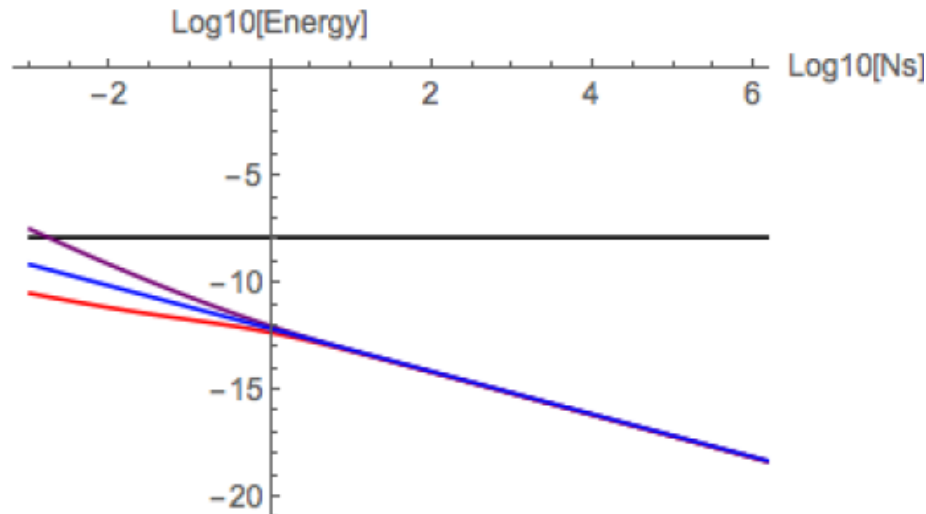
< 1 m ?

# CLASSICAL vs QUANTUM RADAR

Assume some fixed probability of error, e.g.,  $p_{err} = 0.1$  with  $p_{fa} = 10^{-5}$ .

Compare:

- Total Energy  $E$  (in joules) required by a classical radar with  $M = 1000$  pulses (and fixed  $N_s$  photons per pulse)
- Total Energy  $E$  (in joules) required by a quantum radar with variable  $M$  and therefore variable  $N_s$  photons per pulse. Consider entangled/separable/coherent



$$\begin{aligned} \nu &= \frac{c}{\lambda} = 10^9 \text{ Hz} & \eta &= 0.6 & L_{sys} &= F_n = 3 \text{ dB} \\ \chi &= 0.046 \text{ km}^{-1} & d &= 1 \text{ Hz} & \sigma &= 1 \text{ m}^2 \\ T &= 290 \text{ K} & B &= 10^6 \text{ Hz} & G_c &= G_q = 1 \end{aligned}$$

**Distance  $R = 1 \text{ m}$**

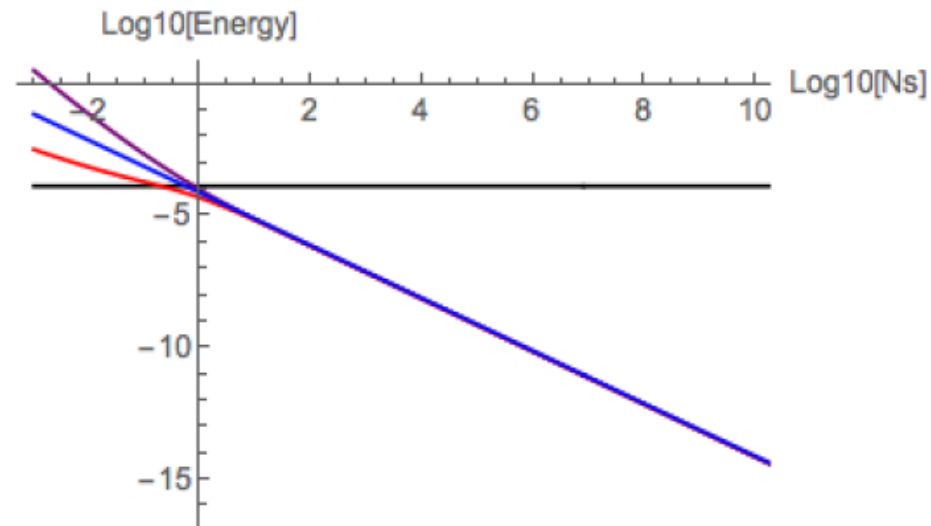


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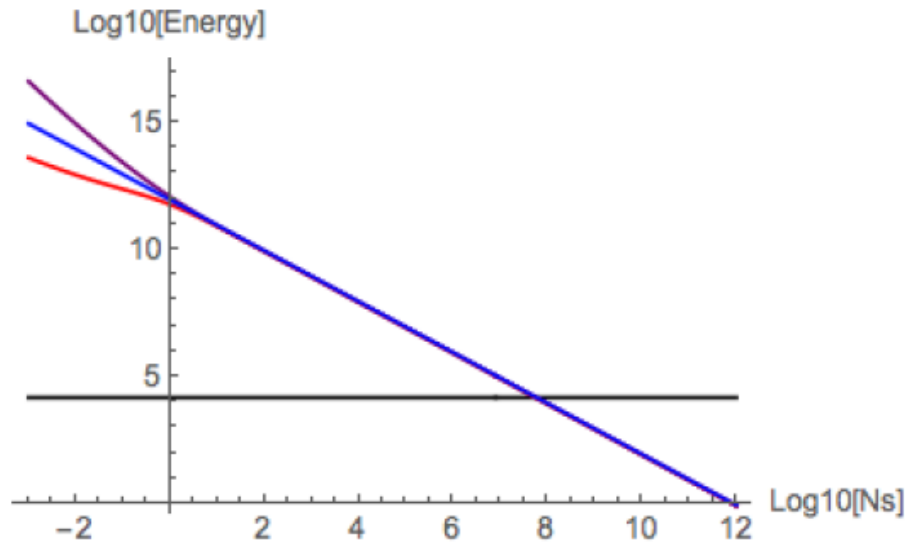
**Distance  $R = 10 \text{ m}$**

# CLASSICAL vs QUANTUM RADAR

Assume some fixed probability of error, e.g.,  $p_{err} = 0.1$  with  $p_{fa} = 10^{-5}$ .

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**Distance  $R = 1000 \text{ m}$**

# CLASSICAL vs QUANTUM RADAR

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Plots show that:

- Separation between entangled sources and coherent states only appears at short ranges  $\leq 1\text{m}$  (for small photons/mode)
- Quantum entangled radar robust to noise but not to loss
- Quantum radar based on coherent states + quantum detection is able to beat the classical radar for range of 1km or so
- Advantage = lowering energy irradiated (low power radar)

So we identify two regimes and two different designs:

- Q. illumination (entangled-based) design for short range
- Coherent-state plus quantum detection for long range

# QUANTUM RADAR as LOW-POWER RADAR

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Low-power quantum radar is interesting for:

- stealthy short-range target detection
- short-range detection in the presence of strong cluttering (thx to its robustness to high background noise)
- proximity sensing and environmental scanning (e.g., drones, cars, airport security...)
- non-invasive quantum microwave spectroscopy, with direct applications to condensed matter physics (solid or atomic spins) and rotational spectroscopy (molecular rotors, organic molecules)
- Eavesdropping of quantum key distribution protocols

# QUANTUM RADAR – next developments?

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Extend the model from yes/no detection to:

- Ranging and Doppler measurements (e.g., to mimic the pulse-Doppler classical radar)
- Tracking

Studies of the performance with respect to:

- Clutter
- Jamming
- etc.

Studies of different configurations:

- Monostatic
- Bi-static
- etc.

THANKS FOR YOUR ATTENTION !