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Quantum Radar: Its Challenges and Potential

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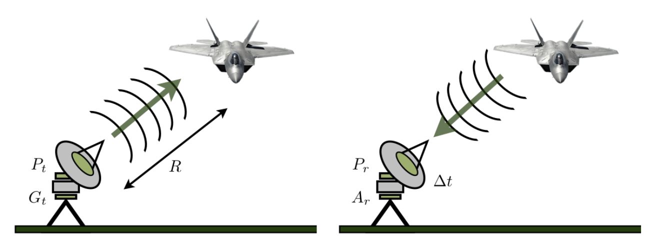
Outline

1. History of quantum radar theory: Quantum Illumination (QI) and extension to the microwave domain using Gaussian states
2. Promised detection advantage
3. Challenges and limitations faced in implementing quantum radar prototypes
4. Overview experimental and theoretical progress in these matters
5. Outline current state and potential for future realisation of a working quantum radar

Introduction

- Quantum technologies has become a vast and highly active field of research: computation, cryptography, sensing, metrology...
- Making use of quantum features (superposition and entanglement) achieve otherwise impossible results, e.g., quadratic improvement in measurement sensitivities ($\propto 1/\sqrt{N} \rightarrow \propto 1/N$)
- Quantum sensing promising in near future applications using non-classical radiation fields and Gaussian state implementation, e.g., *quantum illumination (QI)*
- QI promises to outperform classical counterparts - even when
    - Low reflectivity
    - Low brightness
    - High thermal background

Classical radar

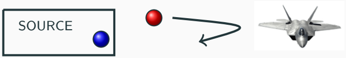
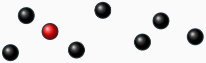
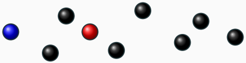


- Classical radar has not fundamentally changed in over 50 years. Obeys the *radar equation*:

$$P_r = \frac{P_t G_t A_r \sigma F^4}{(4\pi)^2 R^4}, \tag{1}$$

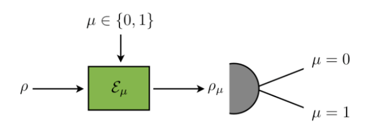
- Introducing noise $P_n = k_B T B_n F_n$, performance depends on SNR$= P_r/P_n$
- Vulnerable to a host of electronic countermeasures

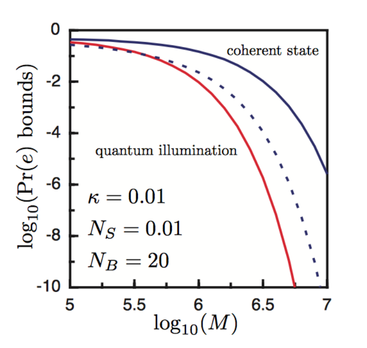
Basic QI protocol

1	Entangle Signal and Idler mode pairs	
2	Send Signal, retain Idler	
3	Collect Return = Noise + Signal (?)	
4	Measure Idler with Return	
5	Detect:	(i) Correlations $\Rightarrow$ Target Present (ii) No correlations $\Rightarrow$ Target Absent

[Lloyd, Science **321**, 1463 (2008)]

Quantum hypothesis testing





- Quantum radar $\equiv$ binary QHT:
  - performance reduced to the distinguishablity of states ρ_μ.
- Requires $\rho_\mu^{\otimes M}$ with $M \gg 1$
- Receiver makes *minimum error-probability decision* between:
  H_0: target absent
  H_1: target present

$$P_{\mathrm{err}}^{QI} = \frac{1}{2} \exp\left(-\frac{M\kappa N_S}{N_B}\right) \qquad (2)$$

$$P_{\mathrm{err}}^{CS} = \frac{1}{2} \exp\left(-\frac{M\kappa N_S}{4N_B}\right) \qquad (3)$$

$(0 < \kappa \ll 1, N_S \ll 1, N_B \gg 1, M \gg 1)$

[Lloyd, Science **321**, 1463 (2008)]
[Tan *et al*, Phys. Rev. Lett. **101**, 253601 (2008)]

Receiver designs

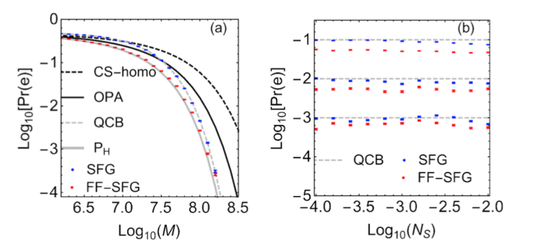
- Previous results assumes use of *optimal* receivers achieving

$$R_{QI} = \frac{\kappa N_S}{N_B} \qquad \text{vs.} \qquad R_{CS} = \frac{\kappa N_S}{4 N_B} \tag{4}$$

- Practical receiver designs include Guha Erkman (2009) - Optical Parametric Amplifer (OPA) and Phase Conjugating (PC) receivers:

$$R_{PC/OPA} \simeq \frac{\kappa N_S}{2 N_B}, \qquad N_S \ll 1, \kappa \ll 1, N_B \gg 1 \tag{5}$$

- Zhuang (2016) FF-SFG receiver (non-linear)



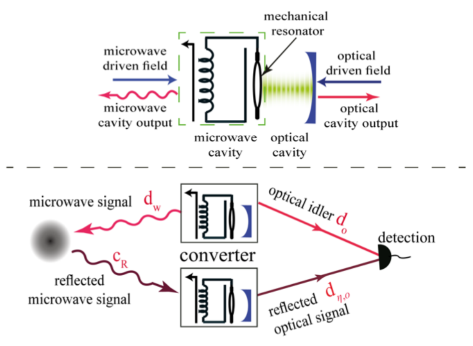
[Guha and Erkmen, Phys. Rev. A **105**, 080504 (2010), Zhuang *et al*, Phys. Rev. Lett. **118**, 040801 (2017), Zhuang *et al*, Phys. Rev. A **96**, 020302(R) (2017)]

Experiments

- Lopaeva *et al*, Phys. Rev. Lett. **110**, 153603 (2013)
  SPDC source and photon-counting correlations to obtain SNR advantage over a correlated thermal state source
- Zhang *et al*, Phys. Rev. Lett. **114**, 110506 (2015)
  Gaussian QI protocol using an OPA receiver (suboptimal). Achieved 20% improvement in effective SNR relative to optimal classical scheme.

Extension to the microwave

- Optical domain, $\sim 10^{15}$Hz, $N_B \sim 0$
- Microwave domain, $\sim 10^{9}$Hz, $N_B \gg 0$!



- Electro-opto-mechanical (EOM) converter uses opto-mechanics to create microwave/photonic entanglement
- Microwave signal as probe
- Optical signals in source generation and detection

[Barzanjeh *et al*, Phys. Rev. Lett. **114**, 080503 (2015)]

Microwave experiments

- Much more challenging! Even with EOM, receiver design is an issue made larger now due to energetics.
- Sandbo Chang *et al*, Appl. Phys. Lett. **114**, 112601 (2019)
- Luong *et al*, arXiv:1903.001001 (2019)
- Barzanjeh *et al*, Sci. Adv. **6**, eabb0451 (2020)
- All three use Josephson parametric converter (JPC) for entanglement generation of microwave modes and compared the performance to a classically-correlated radar.
- Barzanjeh *et al.* additionally showed that their system displayed an effective quantum advantage over the stronger benchmark provided by the use of coherent states though with post-processing.

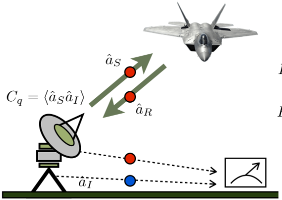
Entangled source?

- Creating large numbers of maximally-entangled photon pairs is demanding!
- Generalise the definition of a quantum radar beyond QI:

  Any model that exploits a quantum part or device to outperform a corresponding classical radar under the same conditions of energy, range, etc.

- We progressively relax entanglement requirements of QI and study the corresponding detection performances to the point where the source becomes just-separable, i.e., a maximally-correlated separable state.

QI with a generic Gaussian state



$H_0 : \hat{a}_R = \hat{a}_B, \quad N_B >> 1$

$H_1 : \hat{a}_R = \sqrt{\kappa}\hat{a}_S + \sqrt{1-\kappa}\hat{a}_B, \quad \kappa << 1, \quad N_B \to \frac{N_B}{1-\kappa}$

Source:

$$\mathrm{V}_{S,I} = \frac{1}{2}\begin{pmatrix} \nu \mathbb{1} & c\mathrm{Z} \\ c\mathrm{Z} & \mu \mathbb{1} \end{pmatrix}, \quad \begin{cases} \mathbb{1} := \mathrm{diag}(1,1), \\ \mathrm{Z} := \mathrm{diag}(1,-1), \end{cases} \tag{6}$$

where $\nu := 2N_S + 1$, $\mu := 2N_I + 1$ and $0 \leq c \leq 2\sqrt{N_S(N_I+1)}$.

Return:

$$\mathrm{V}^0_{R,I} = \frac{1}{2}\begin{pmatrix} \omega \mathbb{1} & 0 \\ 0 & \mu \mathbb{1} \end{pmatrix}, \qquad \mathrm{V}^1_{R,I} = \frac{1}{2}\begin{pmatrix} \gamma \mathbb{1} & \sqrt{\kappa} c\mathrm{Z} \\ \sqrt{\kappa} c\mathrm{Z} & \mu \mathbb{1} \end{pmatrix}, \tag{7}$$

where we set $\omega := 2N_B + 1$ and $\gamma := 2\kappa N_S + \omega$.

What happens when correlations c are not maximal?

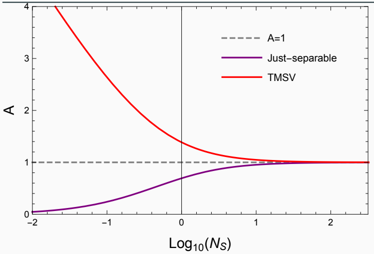
QI with a generic Gaussian state

Symmetric Hypothesis Testing

- Minimising total error probability
- Quantum Chernoff bound
- Upper bound:

$$P_{\text{err}}^{\text{gen}} \leq e^{-M\kappa N_S c^2/N_B c_q^2}/2. \quad (8)$$

- No quantum advantage at separable limit here!



Asymmetric Hypothesis Testing

- Constrain type-I error $P_{\text{fa}} < \epsilon$, allowing more freedom to minimise P_{md}.
- Quantum relative entropy
- Compute error exponent advantage:

$$A(c, N_S) := \frac{c^2}{N_S} \ln\left(1 + \frac{1}{N_S}\right) \quad (9)$$

- Benefits of max-entanglement for QI only for very small energies.
- For increasing N_S, the ratio $A \to 1$, irrespective of source specification.
- Just-separable $\to$ QI at about 20 photons.

[Karsa *et al*, Phys. Rev. Research **2**, 023414 (2020)]

Parameters

- The radar equation relates returning signal power P_R to the transmitted signal power P_T:

$$P_R = \frac{GF^4 A_R \sigma}{(4\pi)^2 R^4} P_T \tag{10}$$

- At the same time, we can also write κ as the ratio:

$$\kappa = \frac{P_R}{P_T} = \frac{GF^4 A_R \sigma}{(4\pi)^2 R^4}, \tag{11}$$

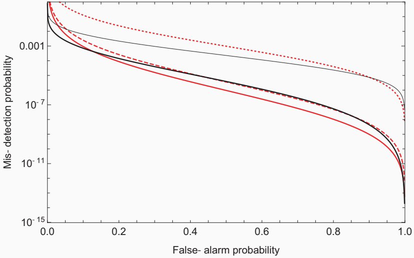
- Assume $F = 1$ (no free-space loss) and ideal pencil beam so that solid angle δ is exactly subtended by the target's σ (valid at short range) so that $G = 4\pi/\delta = 4\pi R^2/\sigma$.
- Then,

$$\kappa = \frac{A_R}{(4\pi R)^2}, \quad R = \frac{1}{4\pi}\sqrt{\frac{A_R}{\kappa}}. \tag{12}$$

[Karsa *et al*, Phys. Rev. Research **2**, 023414 (2020)]

ROC comparisons

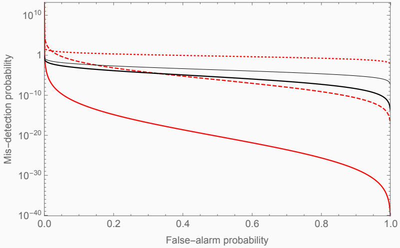
$N_S = 1, R = 1m$



- Red curves: Gaussian state QI with $C(p) = pC_d + (1-p)C_q$. Just-separable, $p = 1$, (dotted), maximal entanglement ($p = 0$), solid, and intermediate correlations ($p = 1/6$), dashed.
- Black curves: Classical coherent state benchmark
  Optimal homodyne detection, thick, and lower bound, thin.

[Karsa *et al*, Phys. Rev. Research **2**, 023414 (2020)]

$N_S = 0.01, R = 0.1m$

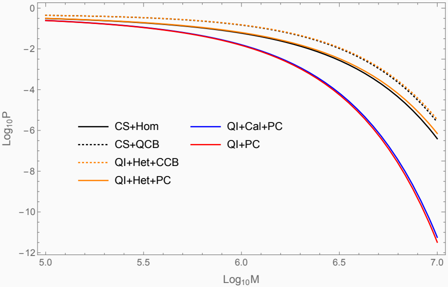


- Red curves: Gaussian state QI with $C(p) = pC_d + (1-p)C_q$. Just-separable, $p = 1$, (dotted), maximal entanglement ($p = 0$), solid, and intermediate correlations ($p = 1/2$), dashed.
- Black curves: Classical coherent state benchmark
  Optimal homodyne detection, thick, and lower bound, thin.

Idler storage and quantum memories

- Idler photon must be stored reliably in a quantum memory.
- Storage must be for time equal to the time of travel for the signal photon (depends on the target's range).
- QI advantage crucially depends on their *simultaneous* joint-measurement.
- Relates to ongoing research on fault-tolerant quantum computation so may be available in the future.
- Experimental proposals for a "quantum noise radar" where signal and idler beams are measured separately, forgoing idler storage needs.

Quantum noise radar? Added noise ϵ



Assume $\varepsilon_{I(R)} = 1$ (heterodyne).

QI+PC: Entangled TMSV source with PC receiver

QI+Het+PC: $\varepsilon_I = \varepsilon_R = 1$ before the PC receiver

QI+Cal+PC: $\varepsilon_R = 1$ and $\varepsilon_I = 0$

$$\mathrm{SNR}_{\mathrm{QI+Cal+PC}} \to \mathrm{SNR}_{\mathrm{QI+PC}} = \frac{(1+N_I)\kappa N_S}{2N_B(1+2N_I)}, \tag{13}$$

$$\mathrm{SNR}_{\mathrm{QI+Het+PC}} \to \mathrm{SNR}_{\mathrm{CS+Hom}} = \frac{\kappa N_S}{4N_B}. \tag{14}$$

The maximal advantage of QI+PC over CS+Hom is given by

$$\frac{\mathrm{SNR}_{\mathrm{QI+PC}}}{\mathrm{SNR}_{\mathrm{CS+Hom}}} = \frac{2(1+N_I)}{1+2N_I} \to 2 \text{ for } N_I \ll 1. \tag{15}$$

[Karsa and Pirandola, IEEE AES Magazine, **35**, 11 (2020)]

Single photon detectors

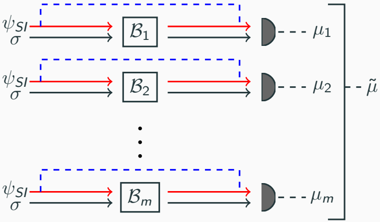
- Fundamentally important: single photon detectors with high detection efficiency.
- Source generation is difficult enough in large numbers (consider also loss due to spherical beam spreading) to lose more at the receiver.
- Especially difficult task in the microwave domain (photon energies are very small).
- Relates to ongoing research in many other quantum technological fields, e.g., requirement for secure quantum key distribution.

Quantum target ranging?

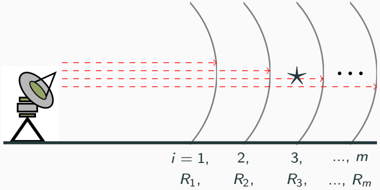
- Protocols for target ranging remain unclear.
- Intrinsically linked to the problem with idler storage.
- When the target's range is unknown, or a measure to be determined, how do you know when to recombine the signal with the idler?
- Meanwhile hoping for lossless storage for the entire duration.
- Potentially "quantum noise radars" could resolve this, but so far their operation has proved sub-optimal.
- Recent developments in quantum-enhanced channel position finding (CPF) could offer a solution.

Quantum target ranging with CPF

Channel Position Finding



Quantum Target Ranging



Sufficient condition:

$$\mathrm{P}^{\mathrm{QTR}}_{\mathrm{CN}} \leq \mathrm{P}^{\mathrm{CTR}}_{H,\mathrm{LB}}, \tag{16}$$

satisfied when

$$\ln m \leq 2M \frac{\kappa N_S}{N_B} \frac{N_B(2-m)+1}{2N_B+1}. \tag{17}$$

For any $N_B > 1$, cannot physically be satisfied for any $m > 2$.

[Zhuang and Pirandola, Comm. Phys. **3**, 103 (2020), Karsa and Pirandola, arXiv:2011.03637 (2020)]

Concluding remarks

- Theoretically, quantum entanglement can offer significant improvements in target detection.
- Barriers towards realisation are largely based on practical engineering issues.
- "Quantum radar" is still very much in its infancy.
- Progress closely relates to that of many other quantum technologies, improved experimental capabilities will no doubt follow.
- Further, quantum radar not limited to using the standard QI source - there are many other aspects of a working quantum radar where the "quantumness" may reside.
- Still a high level of research potential, with potentially very high rewards.

Thanks for listening!