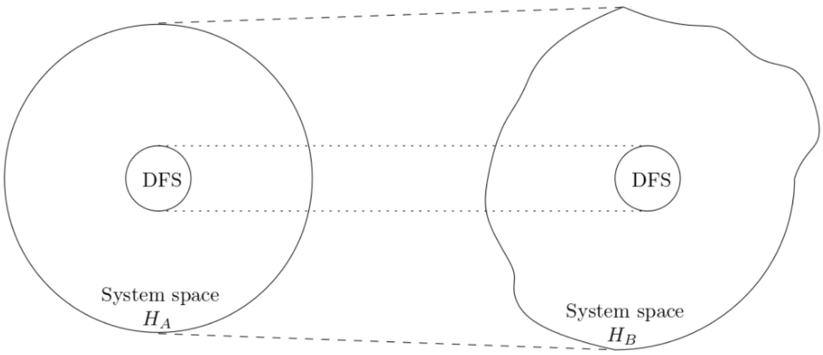
# Stabilizer codes for Open Quantum Systems

The complexity of controlling quantum systems can be reduced by decreasing the noise due to system-environment interaction. This can be achieved by resorting to quantum error-correcting codes. Among them are the stabilizer codes invented by Gottesman. Focusing on the use of stabilizer codes for open quantum systems and quantum metrology, we obtain a new method for improving the measurement accuracy in quantum systems. It is known that quantum systems can be used to obtain a quadratically speed-up in the accuracy of an observable when compared with classical systems. This is named the Heisenberg limit (HL) scaling. However, deriving a systematic approach to achieve HL scaling is a nontrivial task. Previous works using quantum codes rely on the part of the Hamiltonian that is orthogonal to the dissipator part of the open quantum system in order to achieve the HL scaling. We consider a different path. In our method, we use decoherence-free subspaces (DFS) for quantum metrology. Since DFS evolves unitarily, the dissipator part of the open quantum system does not contribute to the evolution. So, analyzing conditions for DFS to behave as stabilizer codes, we show how to HL scaling using decoherence-free stabilizer codes.

## Decoherence-free Stabilizer Codes

Decoherence-free subspaces of the Hilbert space have the effective evolution described unitarily. These subspaces can be used to shield classical or quantum information of noisy environments. However, it is not clear how to construct these subspaces. Connecting DFS and stabilizer codes, we establish a systematic encoding method for DFS by means of the existing encoding algorithms for stabilizer codes. The subspace which is both decoherence-free and stabilizer code is named decoherence-free stabilizer code. Exploring further decoherence-free stabilizer codes, we obtain a new approach to achieve the HL scaling.




In particular, we have obtained the following result: Consider a quantum system with evolution given by a Markovian master equation with Lindblad $\{J_l\}$ operators . Let $\mathcal{S}$ be a stabilizer set constructed from the Lindblad operators. Let $|\psi_{\text{max}}\rangle$ and $|\psi_{\text{min}}\rangle$ be eigenvectors of the system Hamiltonian $H_S$ with maximum and minimum eigenvalues, respectively. Then, Heisenberg limit scaling is achievable if

$$\left|\psi^{(N)}\right\rangle = \frac{1}{\sqrt{2}}(|\psi_{\text{max}}\rangle^{\otimes N} + |\psi_{\text{min}}\rangle^{\otimes N})$$

belongs to the stabilizer code for any $N > N^*$ , where $N^* \in \mathbb{N}$ .

Some of the advantages of our method are

- Decoding algorithms for stabilizer codes can be used in order to efficiently verify if the probing state belongs to the stabilizer code.
- Suppose the probing state satisfies the previous hypothesis, then we can use encoding algorithms to generate the probing state efficiently.

- The classical code corresponding to the stabilizer code can be used as a tool to engineer the environment in order to achieve the Heisenberg limit. This optimization is implemented classically.