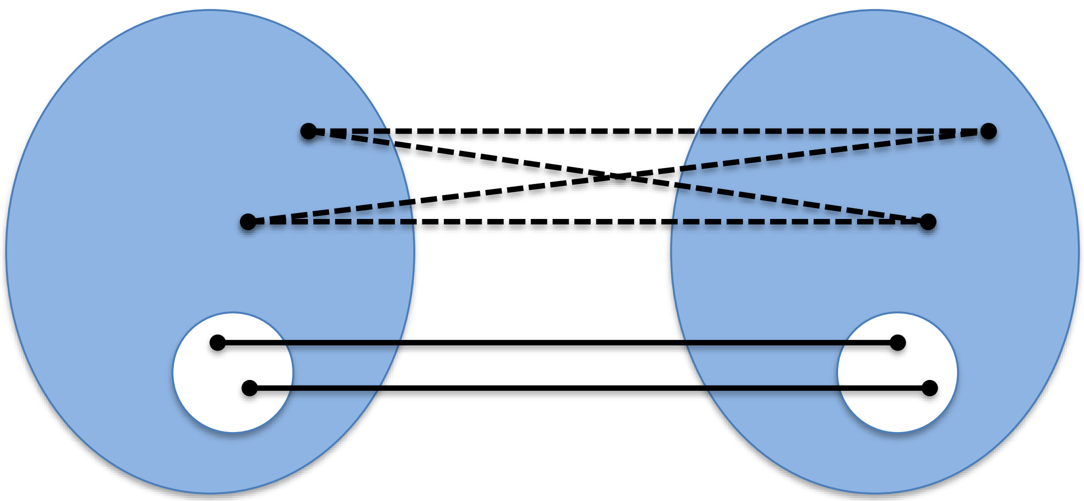
Decoherence-free Stabilizer Codes

A subspaces of $\mathcal{H}$ is named decoherence-free (DFS) when the effective evolution can be described unitarily and free of decoherence



No well known algorithms to encode into DFS. Thus a link to stabilizer code has been provided: Francisco Revson F. Pereira, Stefano Mancini ''Stabilizer codes for Open Quantum Systems'' arXiv:2107.11914

A stabilizer code $\mathcal{Q}_{\mathcal{S}}$ is a subspace stabilized by a set $\mathcal{S}$ of (error) operators

$$\mathcal{Q}_{\mathcal{S}} = \bigcap_{S \in \mathcal{S}} \{|\psi\rangle \in \mathcal{H} : S|\psi\rangle = |\psi\rangle\}$$

Application to Quantum Metrology

Result

Consider a quantum system evolving under a Markovian master equation with Lindblad operators $\{J_l\}$. Let $\mathcal{S}$ be a stabilizer set constructed from the Lindblad operators. Let $|\psi_{\text{max}}\rangle$ and $|\psi_{\text{min}}\rangle$ be eigenvectors of the system Hamiltonian H_S with maximum and minimum eigenvalues, respectively. Then, Heisenberg limit scaling is achievable if

$$\left|\psi^{(N)}\right\rangle = \frac{1}{\sqrt{2}}(|\psi_{\text{max}}\rangle^{\otimes N} + |\psi_{\text{min}}\rangle^{\otimes N})$$

belongs to the stabilizer code for any $N > N^*$, where $N^* \in \mathbb{N}$.

Application to Quantum Metrology

As an example, consider

$$\frac{\partial \rho}{\partial t} = -i[H_S, \rho] + \frac{\gamma}{2}(2J\rho J^\dagger - J^\dagger J\rho - \rho J^\dagger J),$$

with $J = e^r \left(\mathbb{I}^{\otimes N} + \sigma_z^{\otimes N}\right)$, $H_S = \gamma\, \sigma_x^{\otimes N}$ and *r* the squeezing parameter.

The stabilizer set constructed from the Lindblad operator J is given by $\mathcal{S} = \langle (\mathbb{I}^{\otimes N} + \sigma_z^{\otimes N})^i : i = 0, 1\rangle$. Consider the following eigenvectors of H_S

$$\left|\psi_{\text{max}}^{(N)}\right\rangle = \frac{1}{2^{N/2}}(|0\rangle + |1\rangle)^{\otimes N}, \qquad \left|\psi_{\text{min}}^{(N)}\right\rangle = \frac{1}{2^{N/2}}(|0\rangle - |1\rangle)^{\otimes N}.$$

Then we can see that

$$\left|\psi^{(N)}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|\psi_{\text{max}}^{(N)}\right\rangle + \left|\psi_{\text{min}}^{(N)}\right\rangle\right)$$

belongs to the stabilizer code for any $N \geq 1$.

Advantages with respect to DFS:

- Decoding algorithms for stabilizer codes can be used in order to efficiently verify if $\left|\psi^{(N)}\right\rangle$ belongs to the stabilizer code.
- Suppose $\left|\psi^{(N)}\right\rangle$ satisfies the previous hypothesis, then we can use encoding algorithms to generate the probing state efficiently.
- The classical code corresponding to the stabilizer code can be used as a tool to engineer the environment in order to achieve the Heisenberg limit. This optimization is implemented classically.